

ESSAYS IN INDUSTRIAL ORGANIZATION

By

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To my parents

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This dissertation consists of empirical and theoretical essays that examine a variety of incentive problems.

The first essay is an empirical one that examines incentives for collusion in an auction market. It investigates whether bid prices for highway construction are related to conditions that favor the formation of a cartel. The probability of a successful cartel is hypothesized to be higher when there is more repeat bidding (due to lower cost of negotiation), resulting in a higher average price. The empirical results support the prediction. Also, as the number of firms increases, the cost of negotiation increases making cartels less profitable. Consistent with this reasoning, it is found that winning bids fall as the number of bidders rises until there are about 6 to 8 firms. Since subsequent entry has no effect on the winning bid price, it is concluded that the highway construction market becomes competitive once there are about eight bidders.

The other two essays examine a "double moral hazard problem". Double moral hazard exists when the outcome of a production process is influenced by the unobservable actions of both the employer (principal), and the employee (agent). That the jointly efficient outcome is unattainable due to double moral hazard is well known. The main contribution is to show that employment of multiple agents permits contracts that completely or partially resolve the double moral hazard problem. This incentive to employ multiple agents is shown to be independent of the well-known incentive to use multiple agents' outputs to monitor one another when the random components of the outputs are correlated. The use of multiple agents can resolve all incentive problems when there is a moving support on output (essay 2). Introducing another agent whose output also depends on the principal's choice distinguishes shirking by the principal from that of an agent. In the case of a nonmoving support on output (essay 3), although the jointly efficient outcome is not attainable, use of multiple agents does improve the nature of the optimal contract.

## CHAPTER 1

### INTRODUCTION

This dissertation consists of empirical and theoretical essays that examine a variety of incentive problems.

An important potential problem in an auction market is the presence of a bidders' cartel. Previous literature has considered how bidders, by acting collusively in an auction market, can manipulate the winning bid. In the first essay (chapter 2), I consider collusion in the highway construction industry. I examine whether bid prices are related to conditions that favor the formation of a cartel. A better understanding of the determinants of collusion and the impact of collusion on the winning bid price may help buyers and enforcement agencies prevent collusion more effectively.

There is little empirical work in the area of bidding and auctions. Consequently the economic literature has very little evidence on the effect of bid rigging on the winning bid. Existing literature can be divided into two different groups, which vary according to how the presence of a cartel is inferred. The first group consists of studies where it is known whether collusion is present and which firms are colluding. This information is based either on some conviction made in the federal court or on interviews with the suspected colluders. The second group consists of studies where this information on collusion is not available. This group of studies utilizes variables that are associated with



collusion. My study is more closely related to this second group. I propose a technique which relates the existence of collusion to the presence of conditions consistent with implicit collusion (i.e., number of bidders, repeat bidding etc.). Later, I use this inference to test the possible effect of presence of possible bid-rigging on the winning bid, which links my analysis to the analysis of the relationship between market structure and price.

I examine the effect of the presence of collusion on price, i.e., the winning bid, and the bidding behavior of the firms participating in Florida highway construction. First price sealed bid auctions are employed in these projects where the bids are reported fully and correctly, making collusion more effective.

One common way of testing for the presence of collusion is to test the conditions consistent with collusion. As pointed out by Hendricks and Porter (1989), the detection of the presence of collusion in an auction market is not very straightforward. The identification of the existence of collusion depends on the particular auction rules and the collusive mechanism. A cartel can have different collusive mechanisms and bidding patterns. In first price sealed bid auctions, conventionally, identical bids suggest collusion. Undoubtedly, this is a very poor strategy since it would easily alert people about ongoing collusion. The collusive mechanism might also follow a rotating bid pattern where the cartel designates the winner in a particular period, and only that firm submits a bid. This pattern again signals possible existence of collusion. The third type of auction mechanism, which should be prevalent in a procurement auction, is where instead of having identical bids or an alternating pattern of winners, there is one designated winner, and all other firms in the cartel submit pseudo bids to give an

appearance of competition. This is the most credible and unsuspecting means of collusion. In the first essay, I deal with this third type of mechanism, and provide indirect evidence on the possible existence of collusion.

The existing theoretical literature predicts that the extent of collusion will depend on the number and size distribution of firms, firms' technologies and the information structure of the market. The other factor affecting the likelihood of collusion is the repeated interaction amongst the firms since repeated play can enhance profit compared to the outcome of the noncooperative single period game. However, the existing empirical literature provides no evidence on the effect of repeat bidding on the winning bid price. Under the assumption of the existence of phantom bidding, it is evident that repeated interaction between auctioneers might facilitate collusion. In this essay, I test whether repeated interaction is associated with a collusive outcome -- a higher winning bid. I also relate the winning bid to the number of bidders. Both of these provide indirect probabilistic evidence supporting the prediction that collusion raises the winning bid.

This essay also provides evidence on how market structure changes with entry, i.e., at what point competition becomes effective, (and thus collusion inconsequential) in this auction market. I find evidence that this market becomes competitive when there are about eight bidders per project and once the market has between 8 to 9 firms, the next entrant has little effect on the winning bid price.

The second and the third essay (chapter 3 and chapter 4 respectively) of this dissertation address the problem of double moral hazard in principal-agent relationships.

Double moral hazard exists when the outcome of a production process is influenced by the unobservable actions of both the policy designer (the principal) and the worker (the agent). In fast-food franchising, for example, not only need contracts provide franchisees with incentives to provide service, quality, etc., but they also need provide the franchisor with incentives to promote the trademark and innovate. These two essays show how multiple agents can completely resolve (in the case of moving support on output, i.e., when any change in effort level shifts the end points of the output distributions) or improve (in the case where agents' output distributions have nonmoving support) on double moral hazard problems.

The models consider a production process where each agent produces an observable output that depends on the agent's unobserved effort, the principal's unobserved effort, and a random error term. The principal's input is common (public) to all agents as, for example, is national promotion by a franchisor. Under this setting, it is shown that there is gain from making the agents' compensation contingent on each agent's output, regardless of the correlation in the random error terms that affect their outputs.

In the case of a moving output support and when only the agents' actions are unobservable, a single agent can be fully motivated and insured using a discontinuous wage function, with penalization if output falls below a threshold. This penalty scheme is not effective under double moral hazard since the principal will purposely undersupply the input to collect the penalty. Introducing another agent whose output also depends on the principal's choice distinguishes shirking by the principal from that of an agent. This

is because both outputs will be stochastically low in the later case (the contract can be designed in such away that shirking by both agents is not an equilibrium).

On the other hand, in the case of nonmoving supports on agents' output distributions, although the jointly efficient outcome is not attainable, use of multiple agents does improve the nature of the optimal contract. In particular, gains result from inter-linking the agents' wages. This also distinguishes the well-established incentive to employ multiple agents' outputs to monitor one another when random components are correlated from the incentive studied here to engage more workers to correct the problem of double moral hazard in the absence of common uncertainty.

## CHAPTER 2

### AN EMPIRICAL ANALYSIS OF THE EFFECT OF REPEAT BIDDING AND MARKET STRUCTURE ON WINNING HIGHWAY BID

#### 2.1.Introduction

As Stigler (1964) noted, the reporting of bids in first price sealed auctions provides information that facilitates collusion. As a result, bid rigging is an important form of collusion. The bids for a number of services or products involve both large and small projects in different locations. As a result, the group of firms that submit bids varies from project to project and some firms bid together on several different projects. The literature on collusion in bidding does not seem to recognize that collusion is more profitable if the organizational costs associated with reaching an agreement are amortized over more bids. Alternatively, a number of game theory models conclude that firm profits are higher under repeated interaction (i.e., a super game) than under a single period noncooperative game. Collusion is thus more likely to occur when repeat bidding is more prevalent. This paper provides what appears to be the first evidence on this hypothesis.<sup>1</sup> It examines the determinants of winning highway construction bids, using two different measures of repeat bidding. As expected, winning bids are higher when the bidders bid together frequently.

It has long been recognized that a large number of firms creates a competitive environment. But a market with 200 firms may be no more competitive than a market with 100 firms. How many firms are needed to bring about competitive prices? Breshnahan and Reiss (1991) had addressed this question in the retail and professional service markets.<sup>2</sup> They find that the entry of the second and third firm results in more competitive behavior, but once the market has between three and five firms, the next entrant has little effect on competitive conduct.

In the empirical bidding literature, Brannman et al. (1987) find that the winning bid falls as the number of bidders increases. In this literature, there is no evidence yet on how many bidders are required for these markets to be competitive. The results in my paper on highway bidding help to remedy this situation. I find that winning bids fall as the number of bidders rises until there are about 6 to 8 firms. Since subsequent entry has no effect on the winning bid price, I conclude that the highway construction market becomes competitive once there are about eight bidders.

The paper is organized as follows: Section 2.2 presents the model, the predictions, and the research methodology. The empirical model is developed in section 2.3. In Section 2.4, I present the data used for this study and define variables. Section 2.5 discusses the empirical results and Section 2.6 provides concluding observations.

## 2.2 Model, Predictions, and Research Methodology

In this section, I discuss the model I test and the underlying predictions. I study a government procurement auction where the data set describes a market that might

facilitate collusion. The data set shows a concentrated market for large jobs (I take the estimated cost as an indicator of the size of the job) with some bidders interacting in a repeated manner. Moreover, the bids are announced publicly. This makes the detection of violation easy for the members in case any firm deviates from the agreement and undercuts the predecided winning bid.

As will be seen later, in order to utilize a measure of repeat bidding, my data set excludes projects for which there is only one bidder. Thus, I do not study situations in which all firms join a cartel and the cartel designates a winner for each project, which is the only firm to submit a bid for that project. Profits are spread among cartel members through a pattern of rotating bids. The fact that only five percent of the projects in my sample attracted only one bid suggests that this form of collusion is not very common. This is not a surprise, since those procuring these services would quickly become suspicious if they received only one bid on each project. My empirical analysis identifies another form of collusion in bidding: phantom bidding. Here there is once again a designated winner, but other members of the cartel submit higher phantom bids to give the appearance of competition and thus not alert the procurer to the cartel's existence.

### 2.2.1 Number of Bidders

The existing literature on collusion and oligopoly suggests that collusion is more likely in a market where there are fewer competitors (Stigler 1964), i.e., there exists an inverse relationship between the likelihood of collusion and the number of bidders. This is because the greater the number of firms in the industry, the more costly it is for the firms to reach an agreement. Furthermore, punishment by the cartel may be less effective

when cartel members deviate from collusive agreements. In my study, the probability of collusion is expected to be higher in those projects where the number of participants is usually less.<sup>3</sup>

Regression analysis will estimate an average, or "expected," price associated with a given number of bidders (NUMBIDS). This "expected" bid price is a weighted average of the collusive price ( $P_{col}$ ) and of the competitive price ( $P_{comp}$ ).

$$E[P] = Pr_{col} \cdot P_{col} + (1 - Pr_{col}) \cdot P_{comp}$$

where  $Pr_{col}$  equals the probability that the winning bid will be collusive. The collusive bid price obviously is higher than the competitive price, which is assumed not to depend on the number of firms bidding. According to the reasoning outlined above, the probability of collusion falls as the number of bidders rises, causing the average winning bid price to fall. It is also possible that cartels with more members will submit lower winning bids. If so, the fall in  $P_{col}$  as the number of bidders increases also results in a lower expected winning bid price.

If there are enough firms bidding, collusion is never profitable (i.e.,  $Pr_{col}$  equals zero) and the expected winning bid price equals the competitive bid price. Thus, the market is competitive if there are, say, at least  $X^*$  firms bidding. Adding more bidders does not make the market any more competitive; the expected price stays at  $P_{comp}$ .

I capture the relationship just described between the average winning bid price and the number of bidders with a piecewise linear function, known as a "spline", that is depicted in Figure 1 above. When the number of bidders is less than  $X^*$ , the price is inversely related to the number of bidders in the market, and with the entry of the  $X^*$ th



firm, the market becomes competitive.<sup>4</sup> Once the market has  $X^*$  number of bidders, adding more bidders does not affect the nature of competition.

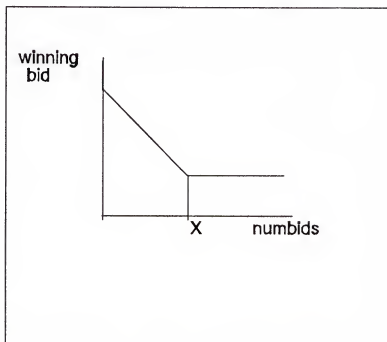


Figure 2.1

THE "SPLINE"

In order to estimate this cut-off point,  $X^*$ , between collusion and competition, I search over different  $X$  for the value  $X^*$ , that gives the best fit (lowest standard error). I introduce two variables,  $BID1X$ , and  $BID2X$ , which are defined in the following way:

If  $NUMBIDS < X$ , then  $BID1X = NUMBIDS$ .

If  $NUMBIDS < X$ , then  $BID2X = 0$ .

If  $NUMBIDS \geq X$ , then  $BID1X = X$ .

If  $NUMBIDS \geq X$ , then  $BID2X = (NUMBIDS - X)$ .

where the term "NUMBIDS" represents the total number of bids in that particular contract.

The above specification clearly portrays two separate segments of the market -- the collusive segment, and the competitive segment. BID1X corresponds to the collusive segment of the market, and BID2X represents the competitive segment. These two variables are used to capture the market size that is required to sustain competition; i.e., they characterize the point beyond which the market is competitive.

### 2.2.2 Repeat Bidding

Another factor signalling the likelihood of collusion under this specified setting is the presence of repeat bids, i.e., the same group of bidders bidding together for different projects. Repeated interaction among the bidders reduces incentives to deviate from the collusive agreement, making successful collusion more likely. This follows from the fact that the deviator would earn a high profit in that particular period of deviation destroying the collusion, and would incur a loss in subsequent periods. Furthermore, with repeated interaction, the cartel's organization costs get amortized over more bids, again making collusion more profitable. This paper focuses on testing this phenomenon empirically. To do so, I construct two indices which capture the occurrence of repeat bidding. I refer to the first proxy as REPEAT1 and the second one as REPEAT2. Both REPEAT1 and REPEAT2 represent a project specific index for the likelihood of collusion.

REPEAT1 is a simple average of the indices  $C(i)$ , where  $i$  ( $i=1, \dots, 667$ ) is the identity of the bidders present in each project. Index  $C(i)$  is defined as the ratio of two

numbers. The numerator is the number of distinct bidders that bidder  $i$  has bid with on all the projects  $i$  has bid on (not joint bids). The denominator is the total number of bids other than  $i$ 's bid on projects that contractor  $i$  has bid on. The higher is  $C(i)$ , the lower is the frequency of repeat bidding for bidder  $i$ . To illustrate the rationale behind  $C(i)$ , consider the following simple example. Suppose firms A, B, C, D, E, K, L, O, and P are bidding on four different projects in the following manner:

TABLE 2.1

Projects	I	II	III	IV
Bidders	A	A	A	D
	B	B	B	K
	C	C	C	L
	D	K	L	O
	E	L	O	P

To construct the index for bidder A,  $C(A)$ , note that A has bid on three different projects. On these three projects, there are twelve other bids in addition to A's bids, and seven other bidders besides A participate in these three projects. Therefore  $C(A)=7/12$ . Likewise, I construct the equivalent indices for all other bidders. They are reported in the Table 2.2.

TABLE 2.2

$$C(A) = C(B) = C(C) = 7/12.$$

$$C(D) = 8/8, \quad C(E) = 4/4.$$

$$C(K) = 7/8, \quad C(L) = 7/12.$$

$$C(O) = 7/8, \quad C(P) = 4/4.$$

A close observation of Table 2.1, and Table 2.2 shows that  $C(i)$  reflects (inversely) the  $i$ 'th bidder's tendency to bid with the same people. For example, consider the indices  $C(A)$ , and  $C(D)$ . The low value of index  $C(A)$  reflects the fact that A has bid with the same bidders (B,C, and L) over several projects. On the other hand, D gets a high index value of 1 because he has not bid together with any same bidder on both his projects. The project index  $REPEAT1(j)$ ,  $j = I, ..IV$ , portrayed below is an average of  $C(i)$ s for the bidders who have bids in that particular project.

$$REPEAT1(I) = (7/12 + 7/12 + 7/12 + 8/8 + 4/4) / 5 = .75$$

$$REPEAT1(II) = (7/12 + 7/12 + 7/12 + 7/8 + 7/12) / 5 = .642$$

$$REPEAT1(III) = (7/12 + 7/12 + 7/12 + 7/12 + 7/8) / 5 = .642$$

$$REPEAT1(IV) = (8/8 + 7/8 + 7/12 + 7/8 + 4/4) / 5 = .932$$

The idea behind these indices is that higher values for the indices should be inversely related to the occurrence of repeat bids. In order to justify my claim, I will demonstrate two extreme cases below. First, consider the case where the same five bidders are bidding in all four projects as follows:

TABLE 2.3

Projects	I	II	III	IV
Bidders	A	A	A	A
	B	B	B	B
	C	C	C	C
	D	D	D	D
	E	E	E	E

Therefore, here  $C(A) = 4/16 = 1/4 = C(B) = C(C) = C(D) = C(E)$ . Consequently, the index for each project is:

$$\text{REPEAT1} = (5/4)/5 = 1/4 = 1/(n-1),$$

where  $n$  is the number of bidders in that project. Next, suppose that there are no repeat bidders in any of these four projects, as in Table 2.4.

TABLE 2.4

Projects	I	II	III	IV
Bidders	A	F	K	P
	B	G	L	Q
	C	H	M	R
	D	I	N	S
	E	J	O	T

Yielding  $C(A) = C(B) = \dots = C(T) = 4/4 = 1$ . Here, the project specific indices are:

$$\text{REPEAT1} = 5/5 = 1.$$

These examples suggest that the higher the project specific index REPEAT1 is (and thus the closer it is to 1), the lower is the incidence of repeat bids, and thus the bids on that particular project are less likely to be collusive.<sup>5</sup>

The second measure REPEAT2 is also a simple average of the indices  $C'(i)$  for each bidder  $i$  in project  $j$ . The index  $C'(i)$  is the ratio of the total number of distinct bidders  $i$  has bid with over the period to the total number of projects he has bid in. Thus, the lower the value of this index, the more likely the firm to be a member of a cartel. Both these proxies give the frequency with which bidders in a project have bid together. To illustrate the construction of  $C'(i)$ s, note that in table I bidder A has bid with seven distinct bidders over three different projects. Thus  $C'(A) = 7/3$ . Similarly I compute this index for all other bidders and they are presented in the following table.

TABLE 2.5

$$C'(A) = 7/3, C'(B) = 7/3, C'(C) = 7/3.$$

$$C'(D) = 8/2, C'(E) = 4/1, C'(K) = 7/2.$$

$$C'(L) = 7/3, C'(O) = 7/2, C'(P) = 4/1.$$

The project specific index REPEAT2( $j$ ), for  $j = I, \dots, IV$ , are constructed and given below.

$$\text{REPEAT2}(I) = \{3.(7/3) + 4 + 4\}/5 = 3$$

$$\text{REPEAT2}(II) = \{3.(7/3) + 7/2 + 7/3\}/5 = 2.6$$

$$\text{REPEAT2}(III) = \{4.(7/3) + 7/2\}/5 = 2.6$$

$$\text{REPEAT2}(IV) = \{8/2 + 7/2 + 7/3 + 7/2 + 4\}/5 = 3.5$$

Now, the higher the number of distinct bidders, the lower the probability of bidder  $i$  to bid with the same group of bidders. To justify this claim, I would again refer to the two extreme cases mentioned above. In the first case, where the same five bidders are bidding in all four projects,  $C'(A) = C'(B) = \dots = C'(E) = 4/4 = 1$ . Hence the project indices are  $5.1/5 = 1$ . In the second case, where there is no repeat bidding,  $C'(i) = 4/1 = 4$ , for  $i = A, B, \dots, T$ . As a result, the project indices are  $5.4/5 = 4$ . So, it is evident that the greater the incidence of repeat bidding (and thus the likelihood of collusion), the lower the value of this index.

### 2.2.3 Size of Project

All my predictions are based on a given cost of performing the work. This cost of course varies from project to project depending on the size of the project. This can be accounted for either by normalizing the bids with respect to the state engineer's estimate of the project's worth, yielding an indicator of the profitability of a specific project, or by taking this estimated cost as an independent variable in the regression. As I will show later, the second specification better conforms to the model.

### 2.2.4 Business Cycle

The ratio of the bid to estimated cost may be higher in economic upturns than in recessions if opportunity costs are not measured very well. Measuring opportunity cost involves inclusion of unobservable costs of production as well as cost of inventories. Therefore correct assessment of economic cost as the business cycle changes is difficult. Cost could be overstated in the face of declining demand and understated during the boom. If so, bids would move procyclically. Besides, there might be a time lag between

estimating these costs and actually observing and declaring the winning bids which might give rise to further inaccuracy.

In contrast to conventional economic theory predicting a procyclical movement of general level of price, recent theoretical and empirical work by Rotemberg and Saloner (1986) suggests that a countercyclical movement in price is more likely in concentrated industries. They argue that implicitly colluding oligopolies are expected to behave more competitively in periods of high demand or boom. This is because during economic upturns or a high demand state, the gain from undercutting is larger and the punishment from deviating is less affected by the state of low demand in subsequent periods.

To test these competing theories, I employ the unemployment rate as a business cycle measure. It is well known that recessions are characterized by high unemployment. As I will see later, my results provide evidence of countercyclical bids, which supports Rotemberg and Saloner's hypothesis.

### 2.3 Empirical Model

To test whether these measures of collusion are related to the price level, I estimate two semilogarithmic models.<sup>6</sup> Here, the regressand is the log of the low bid, and the logarithm of the engineers' estimate of cost is treated as one of the explanatory variables. First, I estimate a simple linear model where the logarithm of the low bid ( $B$ ) is regressed on a measure of repeat bidding ( $REPEAT_j$ ,  $j = 1, 2$ ), and the number of bidders ( $NUMBIDS_i$ ), the unemployment rate in the construction employment sector in Florida ( $UNEMPRT_i$ ), and the logarithm of the estimated cost of the contract ( $E_j$ ). It is



assumed that all low bids follow the same probability distribution. The equation is of the following form:

$$B_i = \hat{\alpha} + \hat{\beta} \text{REPEAT}_{j_i} + \hat{\gamma} \text{NUMBIDS}_i + \hat{\delta} \text{UNEMPRT}_i + \hat{\theta} E_i + \varepsilon_i$$

where  $\varepsilon_i$  is the error term, which has zero expectation and  $\sigma^2$  variance.

This equation is estimated twice: once using REPEAT1, and once using REPEAT2, the two alternative measures of collusion, as an independent variable. The expected sign of the coefficients on REPEAT $_{j_i}$  and NUMBIDS $_i$  should be negative, and that of  $E_i$  should be positive. As I have mentioned earlier, the expected sign of the coefficient estimate of unemployment, the measure of the business cycle parameter could be either negative or positive by two conflicting economic theories. I will show shortly that in this particular study, the parameter estimate is of positive sign.

Then, to test whether there is a discrete piecewise linear relationship between the winning bid and the number of bidders, I estimate the following regression model:

$$B_i = \hat{\alpha} + \hat{\beta} \text{REPEAT}_{j_i} + \hat{\gamma} \text{BIDIX}_i + \hat{\tau} \text{BID2X}_i + \hat{\delta} \text{UNEMPRT}_i + \hat{\theta} E_i + \varepsilon_i$$

#### 2.4. Data

Here I use a data set on Florida Highway building contracts. I analyze the highway construction letting from July 1981 to July 1986.<sup>7</sup> A total of 9065 bids were submitted on 1937 different projects, and there are 667 distinct bidders competing for these projects. The projects having just one bid are eliminated from the statistical work since my measures of repeat bidding (REPEAT1 and REPEAT2) have no meaning when

only one bid is submitted. Omitting these projects and the projects with missing data reduces the sample by 10 percent, leaving 1740 projects with 8952 bids. Descriptive statistics for the variables used in this study are reported in Table 2.6. The first column of this table displays the relevant variables. The last four columns exhibit the mean, the standard deviation, and the minimum, and the maximum values of these variables. This

**TABLE 2.6**  
**DESCRIPTIVE STATISTICS OF THE DATA**

VARIABLES				
	MEAN	STD DEV	MINIMUM	MAXIMU
ESTAMT	2107505.25	5629482.92	7087.02	114575532
BID	2392935.59	7724668.43	3405.11	342371357
WINNING BID	1410398.99	3787554.80	3405.11	73065649
REPEAT1	0.29	0.168	0.0698	1.00
REPEAT2	1.54	1.12	0.355	8.02
UNEMPRT	8.36	1.26	7.00	9.90
NUMBIDS	5.14	2.97	2.00	19.00

NUMBER OF OBSERVATIONS: 8952.

NUMBER OF WINNING BIDS: 1740.

chart indicates that the mean of all submitted bids (BID) is 2,392,936, with a standard deviation of 7,724,668, while the mean winning bid (Winning Bid) is 1,410,399, with a standard deviation of 3,787,555. Therefore, the average winning bid is only 60% as large as the average bid. The mean estimated cost (ESTAMT) for all these projects is

2,107,505, with a standard deviation of 5,629,483. Also, the highway projects attract 5 bids on average, with a maximum of 19 bids in some projects. Repeat bidding appears to be common. With 5 firms bidding on average, REPEAT1 would equal  $1/(5-1) = 0.25$ , if all firms always bid together. The mean value of REPEAT1 is only slightly higher at 0.29. The mean unemployment rate<sup>8</sup> in construction over this period of time is 8.36 with a standard deviation of 1.26.

## 2.5 Empirical Results

This section presents the empirical results of my model. The regression estimates are obtained by ordinary least squares. Table 2.7 reports regressions based on the first specification, which estimates a simple linear relationship between the number of bidders and the winning bid price. The regressions in Table 2.8 are based on the second specification, in which the spline variables (BID2X, BID2X) capture piecewise linear effects of the number of bidders. Each table contains regressions utilizing the full five year sample and utilizing the five year sample without the top and bottom 1 percent of the dependent variable; the latter sample allows me to see how sensitive the results are to the presence of outliers. The two measures of repeat bidding (REPEAT1, REPEAT2) were created using bidding patterns over the five year period (September 1981 to September 1986). To test the explanatory power of these measures in a different time period, Table 2.7 also reports regressions that utilize only the last two years. The OLS results show that coefficient estimates for all parameters have the expected signs. As I

have mentioned earlier, my result indicates a countercyclical movement of price, supporting the Rotemberg-Saloner theory.

The results presented in Tables 2.7 and 2.8 indicate a significantly negative relationship between the winning bids and the two indices of repeat bidding, REPEAT1 and REPEAT2. This supports the hypothesis that there is a direct relationship between the occurrence of repeat bids (and thus the likelihood of collusion) and the market price.

As already noted, my measures of repeat bidding are constructed from bids over the five year period. My use of these measures is predicated on the assumption that the bidding patterns do not change over this period. The fifth and sixth regressions in Table 2.7 examine how sensitive my results are to the time period for the projects (i.e., when the results are based on the past pattern of interaction among the firms). The REPEAT1 and REPEAT2 coefficients for these regressions utilizing the last two years of bids are larger, suggesting that the impact of past patterns of current bids is stronger when repeat bidding has actually occurred (compared to when patterns are merely anticipated); the slightly lower t-statistics reflect the fact that the sample is less than half the size of the full sample.<sup>9</sup>

Let me now examine how big an impact repeat bidding has on the winning bid. The coefficients, although always significantly different from zero, are sensitive to the sample. REPEAT1's coefficients, for example, range from -0.0599 when the extreme values for the dependent variable are deleted (from the five year sample) to -0.1899 when only the last two years are used. For the mean number of bids (5), I have shown that REPEAT1 equals  $.25 [=1/(5-1)]$  when the five bidders always bid together and equals 1

when they never bid together. This potential variation in REPEAT1 ( $.75 = 1 - .25$ ) results, using the smallest coefficient for REPEAT1, in a 4.5 percent variation in the winning bid price.<sup>10</sup>

A simple linear relationship between the number of bids and the price is assumed in the regressions in Table 2.7. The results confirm the expected inverse relationship between the number of bids and the winning bid price. NUMBIDS has negative and quite significant coefficients, which are consistent with collusion and the high bid that accompanies it being less likely when there are more bidders. As explained above, my sample excludes projects that attract only one bidder and thus exclude pure rotating bidding in which a cartel submits only one bid, rotating among the cartel, and no other firms bid. My results suggest that the number of bidders provides some information about market structure. The findings also are consistent with collusion in highway bidding occurring through phantom bidding.

A spline is employed in Table 2.8 to ascertain whether the first price sealed bid auction market becomes fully competitive when there are "enough" bidders and thus additional bidders have no impact on the winning bid. This spline estimates a piecewise linear relationship between the number of bidders and the price. I searched over integers ranging from 3 to 12 for the best value for the break point  $X$ . In the full sample, I get the best fit (i.e., minimum MSE) when  $X$  equals 8. In contrast, in the sample that discards extreme values for the dependent variable, the spline is estimated to change slope when there are 6 bidders.

All the coefficients for BID1X have negative and highly significant coefficients, which suggest that collusion is becoming less prevalent as the number of bidders rises to  $X^*$  (=8 or 6). The estimates based on the full sample imply that the winning bid falls by 12 to 14 percent as the number of bidders rises from 2 to 8. In the truncated sample, the price falls by 9 to 10 percent as the number of bidders rises from 2 to 6.

On the other hand, the coefficients on BID2X show no consistent sign. Using a two-tailed test, two are insignificant, one is significantly positive, and one is significantly negative. It would not be unreasonable to conclude that once there are 8 bidders, additional bidders have no impact on the winning bid. That is, this market appears to become competitive once it has 8 bidders. Notice that my results show that the market becomes competitive at a later point than that found in earlier studies on this subject, where competition has been estimated to exist once the market has three to five firms. Note also that the typical highway project attracts 5 bidders and that my results imply that 8 bidders are needed to have a competitive market. Thus, tacit collusion seems to drive up the price paid for many highway construction projects.

Next, the regression results show that the coefficient estimate of the estimated cost (E) is significantly greater than one. This implies that the elasticity of price with respect to the estimated cost is greater than one. This in turn indicates a higher price-cost margin and thus a higher profit for bigger projects, suggesting that the probability of collusion increases as project size increases.

The significantly positive coefficients on the unemployment rate in Florida construction employment sector imply that highway bids fall as construction activity

TABLE 2.7

## REGRESSION RESULTS

(t-statistics in parentheses)

Variables	FIVE YEARS				LAST TWO YEARS	
	Full Range		Limited Range*		Full Range	
CONSTANT	-0.3615 (-5.920)	-0.3846 (-6.330)	-0.3691 (-8.294)	-0.3801 (-8.578)	-0.5637 (-5.665)	-0.5822 (-5.848)
UNEMPRT	0.0131 (3.565)	0.0133 (3.623)	0.0100 (3.678)	0.0100 (3.700)	0.0333 (5.173)	0.0327 (5.080)
NUMBIDS	-0.0165 (-7.870)	-0.0136 (-6.029)	-0.0168 (-10.921)	-0.0153 (-9.264)	-0.0209 (-5.276)	-0.0174 (-4.116)
E	1.0210 (272.085)	1.0216 (271.711)	1.0225 (372.188)	1.0228 (371.294)	1.0263 (166.152)	1.0263 (166.152)
REPEAT1	-0.1284 (-4.008)		-0.0599 (-2.530)		-0.1899 (-3.608)	
REPEAT2		-0.0264 (-4.380)		-0.0131 (-2.933)		-0.0353 (-3.518)
R-SQUARE	0.9782	0.9783	0.9885	0.9885	0.9759	0.9759
ADJ R-SQ	0.9782	0.9782	0.9885	0.9885	0.9758	0.9757
SSE	81.2011	81.0620	41.6602	41.6085	34.7149	34.4444
MSE	0.0450	0.0449	0.0235	0.0235	0.0469	0.0469
# of Observations	1740		1707		740	

\* Top and Bottom 1% of the Dependent Variable Eliminated.

TABLE 2.8

## REGRESSION RESULTS FOR THE "SPLINE"

(t-statistics in parentheses)

Variables	FIVE YEARS			
	Full Range		Limited Range <sup>*</sup>	
CONSTANT	-0.3633 (-5.793)	-0.3835 (-6.153)	-0.3667 (-7.939)	-0.3757 (-8.177)
UNEMPRT	0.0150 (4.008)	0.0149 (4.020)	0.0114 (4.195)	0.0114 (4.168)
BID16			-0.0242 (-8.291)	-0.0229 (-7.722)
BID26			-0.0065 (-2.205)	-0.0045 (-1.469)
BID18	-0.0236 (-7.976)	-0.0208 (-6.780)		
BID28	0.0079 (1.294)	0.0122 (1.974)		
E	1.0217 (268.422)	1.0224 (268.443)	1.0231 (368.160)	1.0235 (367.700)
REPEAT1	-0.1212 (-3.728)		-0.0564 (-2.352)	
REPEAT2		-0.0274 (-4.508)		-0.0143 (-3.160)
R-SQUARE	0.9785	0.9786	0.9887	0.9887
ADJ R-SQ	0.9784	0.9785	0.9887	0.9887
SSE	77.8958	77.6105	39.6405	39.6405
MSE	0.0449	0.0447	0.0233	0.0233
# of Observations	1740		1707	

\* Top and Bottom 1% of the Dependent Variable Eliminated.



surges. As I have already mentioned, this countercyclical movement is inconsistent with costs being overestimated in downturns. It does, however, support the Rotemberg-Saloner theory and provides evidence that collusion is more susceptible to breaking down during a high demand state, leading to a lower price during a boom. This is an interesting finding since the construction industry is well known to be procyclical. Clearly, this provides added evidence in favor of existence of collusion in this highway auction market since in a competitive market, high unemployment or recession would result in lower costs and thus lower bids if the estimated costs do not capture this cyclical cost fluctuation. The positive relation found in this study is consistent with collusion being more effective during downturns.

## 2.6 Conclusions

This paper presents evidence consistent with collusion sometimes being found in the highway construction auction market. It is the concept of repeated interaction amongst the bidders that motivates this study, and I test the impact of repeat bidding on winning bid prices. My finding that bid prices are higher when repeat bidding is more prevalent supports the hypothesis that repeat bidding is conducive to collusion. Similarly, the fall in bid prices as the number of bidders increases implies that collusion is less common when there are more bidders. Thus the number of bidders provides some information to procurement agencies. Indeed, my result suggests that collusion is normally nonexistent in these markets if there are at least eight bidders. My other

findings indicate that collusion is more prevalent in large projects and when business is slow.

Unfortunately, one potential problem in using repeated interaction among bidders as an indicator of collusion is that the repeated interaction itself might be an endogenously determined factor. Possible factors that might influence the incidence of the same groups of firms bidding together include the geographical area of the projects, the size and type of the project, or other similarities between projects. This can be explained by the fact that it is more likely that the same firms will bid on similar types of projects, (e.g. adding a lane, constructing a bridge etc.). Also, it may be easier to collude if the firms are located in the same area since the participants should be few enough for effective and efficient coordination. Accordingly, one could use the number of bidders per area instead of using the number of bidders per project to measure collusion. More importantly, it would be interesting to see what actually determines the participation and the number of bidders per project in such a strategic dynamic setting. The data at this time do not permit me to explore these questions, but this would be a good topic for future research.

### Notes

1. Much of the empirical literature on bidding utilizes information on collusion that is based on interviews or on judicial convictions. See, for example, papers by Comanor and Schankerman (1976); Feinstien, Block, and Nold (1985); Porter and Zona (1992); Froeb, Koyak, and Werden (1993). Interviews and convictions do not, however, uncover every cartel since colluders do not always reveal their activities and not all colluders are discovered and convicted. I avoid this error in measuring cartel activity by relating measures of the propensity to collude to the

price. Others [McClave, Rothrock, and Ailstock (1978); Zona (1986)] have focused on developing methodologies for identifying collusion.

2. See also studies by Lane (1989), who considers this problem in an ATM manufacturers' market, and by Reiss and Spiller (1989), who examine the small airline industry.
3. A project might attract few bidders because of the fact that the project is in an area with infrequent action or because of its size. Fewer bids on large projects might arise because larger projects require more resources, and few firms have the required capacity.
4. For simplicity, while modelling this criteria, I am assuming that a discrete cut-off point between collusion and competition exists, i.e., price no longer varies with the number of bidders beyond a certain number of bidders. In reality the number of firms in the market may have a nonnegligible effect on price over the relevant ranges. This would imply a nonlinear relationship between the winning bid and the number of bidders in the project. I test for this possibility by considering a quadratic model where in addition to all the other explanatory variables, the log of the winning bid is a function of NUMBID and NUMBID<sup>2</sup>. This, however, gives a worse fit as compared to the piecewise linear case described in the text. The regression results with the quadratic model specification suggest that the price reaches a minimum when the number of competitors in the market is 10. The corresponding number is 6 to 8 in the linear model. In the quadratic case, there is a 14 percent decrease in the logarithm of the winning bid as the number of bidders increase from 2 to 10. For the eleventh bidder, there is just a .003 percent change. Also, the quadratic specification yields a 1 percent fall in the logarithm of the bid when the number of competitors increase from 8 to 10. In addition to this, I have also tried a three segment linear specification. Setting the number of bidders for the second cut-off at 10, 11, or 12, the third segment is insignificant (positive slope). This suggests that the two segment linear functional form I employ is a reasonable approximation.
5. In this context, note that the underlying assumption is that when a cartel decides to bid for a particular project, all cartel members participate and submit bids. But there is a possibility that the cartel is rotating the bids, i.e., the bidders are intentionally dividing up the market. Clearly, in such a case, my indices would be uncorrelated with the expected winning bid price.
6. I tried estimating two different models, the semilogarithmic one, and the linear one. The parameter estimates obtained from both models give the predicted signs for the coefficients. However, I found a considerably better fit for the semilogarithmic one and accordingly, only the results obtained from this regression have been reported in this paper. In the simple linear model, the

normalized low bid (i.e., the winning bid /cost) is regressed on the same independent variables except for the logarithm of the estimated cost. In that case, the equation to be estimated is of the following form:

$$b_{wt} = \alpha_1 + \beta \text{REPEAT}_t + \gamma \text{NUMBIDS}_t + \delta \text{UNEMPRT}_t + \varepsilon_t$$

Where  $b_{wt}$  is the normalized low bid for job  $t$ .

Note that the assumption here is that the elasticity of price with respect to cost is 1, i.e., there is an equiproportional increase in price when the cost increases. This obviously leaves the profit margin and the price-cost margin unchanged. This, however, does not correspond to the features of a concentrated oligopolistic market. This leads me to estimate the other model. Clearly, the semilogarithmic model does not make any assumption about the elasticity of price with respect to cost and as will be seen later, this elasticity is significantly greater than one implying that the linear specification is not supported by the data. The semilogarithmic specification also conforms to the customary collusion theory indicating a higher price-cost margin for larger bids, where collusion may be more common.

7. I had access to the data set only over this time period.
8. I also tried using the Florida unemployment rate and the US construction sector unemployment rate to measure the business cycle. The construction sector unemployment rate in Florida is reported in the text since that is more pertinent for this study. The parameter estimates for the other two unemployment measures indicate the same countercyclical movement in price, and the results are significant.
9. The coefficients for REPEAT1 and REPEAT2 are, however, insignificant (although they have the predicted signs) when just the fifth year is utilized. The insignificance could be due to the comparatively small sample size (one ninth of the full sample), which produces less precise estimates. The other explanation is based on the observation that the last year was a time of economic upturn, in which collusion was less common (see my discussion of the unemployment rate), and thus its effects more difficult to detect.
10. The logarithm of the bid price rises by 0.0449  $[(0.75) \cdot (0.0599)]$ .

## CHAPTER 3

### MONITORING THE PRINCIPAL WITH MULTIPLE AGENTS

#### 3.1 Introduction

Double moral hazard arises in the principal-agent model when both parties provide a non-verifiable input following contracting. Recent empirical investigation (Lafontaine, 1992) attests to the relevance of this environment, and economists continue to analyze it (Carmichael, 1983; Eswaran and Kotwal, 1985; Demski and Sappington, 1991; Itoh, 1992; Bhattacharya and Lafontaine, 1992; and Romano, 1994). The fact that the best feasible contract can engender only a second-best outcome quite generally is well known (Holmstrom, 1982; Kambhu, 1982). The output of the production process fails to permit the contract to distinguish agent shirking from principal shirking.

In many cases of double moral hazard, the principal's input will be public to multiple agents. The franchisor's choice of marketing and/or innovation expenditures will affect the demands faced by multiple franchisees. The sales manager's provision of market information and sales training may effect the success of numerous salespersons. The producer's choice of a durable good's quality will impact multiple consumers' utilities, along with their own care in usage. Finally, the teacher's choice of exam difficulty will affect the performance of an entire class of students. This paper shows that

such an environment often permits attainment of a first-best outcome. The ability to condition agent rewards on multiple outputs permits a contract that disciplines simultaneously both the agents and the principal.

I study the double-moral-hazard problem in a fairly general but simple model which yields very sharp results. The principal's input enters two agents' production functions as a weak complement, and there are moving supports on outputs. I show a contract exists that engenders a unique Nash equilibrium with first-best input choices by all parties. This is in spite of the fact that the incentive problems preclude a first-best outcome in the analogous setting with only one agent employed.<sup>1</sup> Since the contract I present will not penalize the agents when dually low outputs suggest shirking by the principal, a natural concern for collusive exploitation by the agents arises. Cannot the agents agree to both shirk, undermining the contract at the principal's expense? I show further that contracts exist which are also coalition proof (as defined by Bernheim, Peleg, and Whinston, 1987).

I start by presenting the related literature in Section 3.2. Section 3.3 describes the model, and the results are reported in Section 3.4. Section 3.5 contains the conclusions.

### 3.2 Related Literature

This paper straddles the two strands of the incentives literature on multiple agents and double moral hazard. The key insight in the multiple agents literature on moral hazard is that correlation across agents in the randomness affecting outputs permits gains by basing each agent's reward on all agents' outputs (Holmstrom, 1979 and 1982; Lazear

and Rosen, 1981; Green and Stokey, 1981; Nalebuff and Stiglitz, 1983; Mookherjee, 1984; and Ma, 1988).<sup>2</sup> The motivation for inter-linking agents' rewards in this paper derives from principal moral hazard, a fundamentally distinct phenomenon.<sup>3</sup> The results here are independent of whether correlation in random effects on agents' outputs is present. At a more abstract level, the two contracting problems are closely related, both contracts exploiting information gleaned from the set of outputs about the respective sets of decisions in each case.

The subset of papers on multiple agents that assesses the performance of rank-order tournaments warrants particular mention, since one motivation for its study is its correction of another type of potential principal moral hazard (Bhattacharya, 1983; Malcomson, 1984 and 1986).<sup>4</sup> When rewards depend on output levels, the principal has an incentive to misrepresent outputs to reduce total wages. Court enforcement of such contracts then requires that outputs are verifiable. Reward structures that depend only on rank order remove this potential hazard, because total wages do not vary with outputs. Since a rank order will always exist, court enforceability requires only that wage payments are verifiable. I assume that outputs are verifiable, eliminating this potential hazard as an issue.<sup>5</sup>

Most of the literature on double moral hazard has been concerned with the nature of second-best contracts. In addition to the papers cited above, this includes several analyses of warranties (Cooper and Ross, 1985 and 1988; and Mann and Winnick, 1988). A few papers have topics more closely aligned with mine. Demski and Sappington (1991) show how a buyout option in a contract can sometimes resolve double moral

hazard. Their model requires that, following contracting, the principal's choice is made first and observed by the agent. My model presumes that actions are taken simultaneously, rather than sequentially. Carmichael (1983), Itoh (1992), and Bhattacharyya and Lafontaine (1992) analyze principal-agent models with double moral hazard and with multiple agents. Bhattacharyya and Lafontaine intend to rationalize some stylized facts about franchising, using a model that exogenously precludes agent-interdependent reward structures. Itoh considers the issue of task assignments. The technology permits flexibility in who does what, and double moral hazard might arise endogenously. His analysis does not go beyond Carmichael's with regard to the issue of resolving double moral hazard.<sup>6</sup> Carmichael's paper is closest to mine. He is the first (to my knowledge) to describe the incentive to inter-link agent's rewards to alleviate double moral hazard. His analysis is limited by assuming a particular production function, symmetric agents, and by requiring agent rewards that are linear in outputs. The latter restriction is quite significant as it implies the first-best outcome is attainable only when no risk or no agent risk aversion is present. Further, Carmichael does not consider the issue of coalition proofness.<sup>7</sup>

### 3.3 The Model

The principal employs two agents. Let  $e_p \in [0, \infty)$  denote the productive effort of the principal and  $e_i \in [0, \infty)$  that of agent  $i$ ,  $i = 1, 2$  (the latter is not restated when it is clear by context). Output  $i$  is determined by the differentiable production function  $q_i =$



$f(e_i, e_p, \theta_i)$ , where  $\theta_i$  is a continuous random variable with support  $[\theta_{\min}^i, \theta_{\max}^i]$ . Assume  $f(0, 0, \theta_i) = 0$ ; and, for at least one of  $(e_i, e_p)$  positive,

$$(A3.1) \quad f_{e_i}^i > 0; f_{e_p}^i > 0; f_{\theta_i}^i > 0; \text{ and } f_{e_i e_p}^i \geq 0;$$

where subscripts (on functions) denote the respective partial derivatives. Weak complementarity of the principal's and agent's inputs ( $f_{i2}^i \geq 0$ ) helps eliminate multiple equilibria under the contract I study and, of course, characterizes many standard production functions (e.g. the CES). The random elements affecting production may or may not be correlated. I assume the joint density of  $(\theta_1, \theta_2)$  is everywhere positive over  $[\theta_{\min}^1, \theta_{\max}^1] \times [\theta_{\min}^2, \theta_{\max}^2]$ , implying imperfect correlation and a more interesting problem.

Agent  $i$ 's twice differentiable utility function is given by  $U^i = U^i(w_i, e_i)$ , where  $w_i$  is the monetary compensation from the principal. Assume:

$$(A3.2) \quad U_{w_i}^i > 0; U_{w_i e_i}^i < 0; \text{ and } U_{e_i}^i < 0.$$

Hence, effort is costly and the agents are risk averse. Let 0 equal agent  $i$ 's reservation utility.

The principal is risk neutral and has effort cost  $C = C(e_p, n)$ , where  $n \in \{1, 2\}$  is the number of agents employed. Of course,  $C_1 > 0$ . Cost economies from employing two agents may or may not be present, e.g.  $C(e_p, 2)$  may equal  $2C(e_p, 1)$ . For example, the bulk of a franchisor's (principal's) marketing expenditures may entail renting space in local newspapers, and thus increase proportionally with the number of geographically dispersed franchisees (agents). The important "public property" of the principal's input is that it is invariant across the two agents. To sharpen the focus on incentive effects, I assume only

that it would be weakly optimal to employ two agents in the first-best arrangement.<sup>8</sup> The first-best problem (FBP) is:

$$\begin{aligned} \text{MAX}_{e_1, e_2, e_p, w_1, w_2} \quad & E[f^1(e_1, e_p, \theta_1) + f^2(e_2, e_p, \theta_2)] - w_1 - w_2 - C(e_p, 2) \\ \text{s.t.} \quad & U^i(w_i, e_i) \geq 0, \quad i = 1, 2; \end{aligned} \quad (\text{FBP})$$

for values  $w_i$ , since provision of complete insurance is efficient. Let  $e^*$  indicate a solution to (FBP). I assume that the solution to (FBP) is unique with  $e^* \equiv (e_1^*, e_2^*, e_p^*)$  interior. I also assume that either agent can, at finite cost, guarantee the minimum feasible output at the first-best choices in the event the principal chooses  $e_p = 0$ :

$$(A3.3) \quad \exists \hat{e}_1 \text{ s.t. } f^1(\hat{e}_1, 0, \theta_{\min}^1) = f^1(e_1^*, e_p^*, \theta_{\min}^1) \text{ with } U^1(w_1^*, \hat{e}_1) > -\infty.$$

(A3.3) will be used to break "grim choices," e.g.  $e = (0, 0, 0)$ , as an equilibrium at the optimal contract.

The timing and information structure in the problem of interest is now described. Effort choices are non-observable, hence, are, in effect, made simultaneously. The resulting outputs are observable and verifiable. Wages can then be conditioned on outputs but not efforts:  $w_i = w^i(q_1, q_2)$ . A contracting stage precedes a choice-payoff stage. In the contracting stage, the principal offers wage functions to the agents, who can accept or refuse the offer. If either refuses, the game ends. Assuming play proceeds to the next stage, efforts that make up a simultaneous-move Nash equilibrium, conditioned on the contract, are made. Outputs are realized, along with the implied payoffs. The budget is balanced so the principal's payoff is the residual.

### 3.4 Results

A benchmark problem has the principal employ only one agent. A proof that the corresponding first-best outcome is unattainable is in the appendix.<sup>9</sup> The argument is a minor extension to my case of uncertainty of Holmstrom's demonstration (1982, Theorem 1, p 326), in a model without uncertainty, that moral hazard in teams is unresolvable. The intuition is analogous: With a balanced-budget restriction, no enforceable contract exists that can eliminate simultaneously externalities in the principal's and agent's choices. I show by construction that employment of two agents permits complete resolution of the problem.

I construct a variant of a "forcing contract" and then show it supports the first-best outcome. To specify the wage functions, I first describe a partitioning of output space:  $q = (q_1, q_2)$ . Let  $g(q; e)$  denote the joint density function of  $q$ , given the effort choices  $e = (e_1, e_2, e_p)$ . Let  $S(e) \equiv \{q \in \mathbb{R}^2_+ | g(q; e) > 0\}$  denote the support of  $q$ . Using (A3.1), note that  $S(e)$  is a rectangle in output space.<sup>10</sup> Define the following regions (sets), illustrated in Figure 3.1.

$$IA \equiv \{q \in \mathbb{R}^2_+ | q_i \geq f^i(e_i^*, e_p^*, \theta_{\min}^i), i=1,2\}.$$

$$IB \equiv \{q \in \mathbb{R}^2_+ | q \in S(e) \text{ and } e_p \in [0, e_p^*]\}.$$

$$I \equiv IA \cup IB.$$

$$II \equiv \{q \in \mathbb{R}^2_+ | q \notin I \text{ and } q \in S(e) \text{ with } e_1 \geq e_1^*, e_2 \leq e_2^*, \text{ and } e_p \in [0, e_p^*]\}.$$

$$III \equiv \{q \in \mathbb{R}^2_+ | q \notin I \text{ and } q \in S(e) \text{ with } e_1 \leq e_1^*, e_2 \geq e_2^*, \text{ and } e_p \in [0, e_p^*]\}.$$

$$IV \equiv \{q \in \mathbb{R}^2_+ | q_i < f^i(e_i^*, 0, \theta_{\min}^i), i=1,2\}.$$

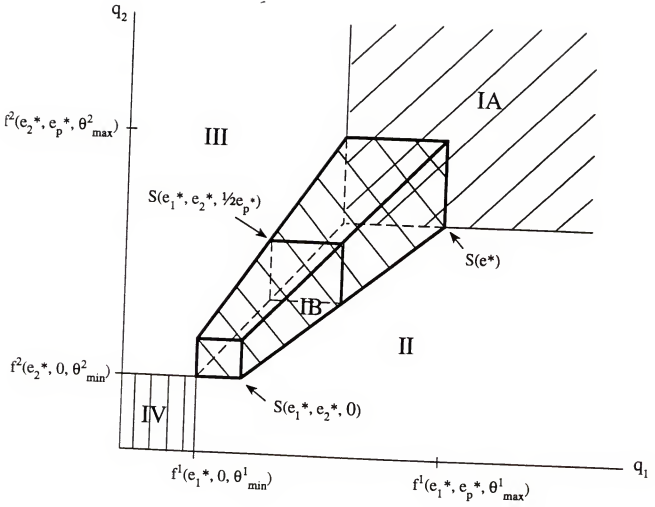


Figure 3.1

## THE PARTITIONED OUTPUT SPACE

The wage functions I study are of the form:

$$(w^1(q), w^2(q)) = \begin{cases} (w_1^*, w_2^*) & \text{if } q \in I; \\ (w_1^* + R, w_2^* - R) & \text{if } q \in II; \\ (w_1^* - R, w_2^* + R) & \text{if } q \in III; \text{ and} \\ (w_1^* - R, w_2^* - R) & \text{if } q \in IV; \end{cases} \quad (WF)$$

where  $w_i^*$  is part of the solution to (FBP). The rewards to the agents are constant over region I, and, in particular, over region IB. Region IA contains the feasible output realizations should all parties put forth at least the first-best efforts. Note how region IB is constructed. It is the "shaft" of feasible output realizations given the agents choose first-best efforts but the principal weakly shirks. Constancy of wages over this region is key to correcting principal moral hazard. Region II is made up of the feasible set of output realizations, not already in region I, having the principal and agent 2 weakly shirk (relative to the first-best efforts), and agent 1 put forth at least the first-best effort. The penalty the contract imposes on agent 2 in region II is key to correcting agent moral hazard. Region III reverses the roles of agents 1 and 2 relative to region II. The symmetric rewards to the harder working agent in the latter two regions are to prevent agent coalitions. Region IV contains the remaining output realizations the model admits, feasible only if every party shirks.

Proposition 3.1

For  $R$  sufficiently high, (WF) supports  $e^*$  as the unique Nash-equilibrium choice vector.

Proof  $e^*$  is a Nash equilibrium. Any deviation by the principal from  $e_p = e_p^*$  keeps  $S(e)$  in region I, over which the principal's payments to the agents are invariant. Hence, the principal chooses  $e_p$  to maximize the difference between expected revenues and his effort costs, i.e. solves the first-best problem. Then  $e_p = e_p^*$  is his best response.

Consider agent 1. By construction, the choice of  $e_1 = e_1^*$  as a response would give the agent her reservation utility. Any choice of  $e_1 > e_1^*$  would keep  $S(e)$  in region IA, with a constant wage but higher effort cost. Any choice of  $e_1 < e_1^*$  would cause  $S(e)$  to overlap or be contained in region III, with a positive probability of imposition of the penalty,  $R$ . For  $R$  sufficiently high, then,  $e_1^*$  is the best response. A symmetric argument applies to agent 2.

The proof of uniqueness is straightforward but tedious, and is in the appendix.

Q.E.D.

Double moral hazard in the presence of one agent is not resolvable because the observation of a low output does not permit agent shirking to be distinguished from principal shirking. Observation of two outputs, both depending on the principal's input, permits identification of a culprit. More precisely, it permits unambiguous identification of a shirking agent with positive probability, thereby allowing sufficient punishment to deter shirking. For the moment, suppose at most one player shirks. The observation of a low  $q_2$  but not low  $q_1$ , i.e. observation of  $q \in II$ , would imply shirking by agent 2. Heavy punishment of agent 2 in this region keeps her from shirking. Likewise, (WF) prevents unilateral shirking by agent 1. The principal's incentive to shirk unilaterally is controlled simply by keeping his wage bill constant over the space of outputs consistent with principal shirking, i.e. over the shaft IB. The agents are not punished if both outputs

are low but sufficiently close in magnitude. As the residual claimant then, the principal has non-distorted incentives.

The possibility that more than one player shirks encompasses two issues. Less interesting is the possibility of multiple Nash equilibria under the contract. Related problems sometimes admit equilibria where one or some players shirk, and the other player or players pick up the slack to prevent punishment [e.g. see Arrow's (1985) discussion of Holmstrom (1982)]. Here the concern is for having the principal shirk and both agents work harder to keep  $S(e)$  completely in region I. Consider, e.g. the possibility of such an equilibrium with the lower left corner of  $S(e)$  aligned with the lower left corner of region IA. The weak complementarity of principal and agent inputs precludes such equilibria. With a constant total wage bill over regions I, II, and III, the principal has efficient incentives to choose effort. Higher than first-best agent efforts would then induce no lower principal effort, because of weak complementarity. Also, as noted above, (A3.3) helps break grim choices with everyone shirking as equilibria. Details of these arguments are in the appendix. Note, however, that  $e^*$  being an equilibrium is independent of weak complementarity and (A3.3). Dropping these assumptions, one might defend  $e^*$  as the focal equilibrium if multiple equilibria exist.

More interesting is the potential for coalition formation to exploit the contract. The contract seems to invite collusive shirking by both agents that would move  $S(e)$  down the shaft, while maintaining their wages with lower efforts.<sup>11</sup> Proposition 2 shows, however, that the Nash equilibrium (WF) engenders is coalition proof (Bernheim, Peleg, and Whinston, 1987). For three-player games, a Nash equilibrium is coalition proof if and only if the following holds. For any two players, their equilibrium payoffs weakly Pareto dominate their payoffs in the set of Nash equilibria among those two players that

take the third player's (the outsider's) action as given. The notion is that the only actions feasible to a coalition of players are those that are equilibrium actions among the subset comprising the coalition (and taking the coalition outsiders' actions as given).<sup>12</sup> This makes sense, since, otherwise, one gives a deviating coalition greater powers of enforcement than has the entire set of players (the coalition of the whole).

Proposition 3.2 For  $R$  sufficiently high, the Nash equilibrium engendered by (WF) is coalition proof.

Proof I first reject agent-agent coalitions. Given  $e_p = e_p^*$ , such a coalition would have to have  $S(e)$  contained in region I, with at least some overlap with region IB. Otherwise, for  $R$  sufficiently high, at least one agent would be worse off than with  $e = e^*$ . But neither can the former choices be equilibrium ones among the agents, since either could gain by increasing her effort. For example, given  $e_2 < e_2^*$ , agent 1 could increase effort to  $e_1 = e_1^*$ , causing  $S(e)$  to overlap with region III, and thereby receive a bonus of  $R$  with positive probability.

Now consider an agent-principal coalition, say with agent 1. Keep in mind that  $e_2 = e_2^*$ . I need not consider choices that have  $S(e)$  overlap with region IV since agent 1 would be worse off than when  $e = e^*$ . Since the total wages the principal pays are constant over regions I, II, and III, the only way that the principal and agent can have payoffs that Pareto dominate the Nash equilibrium ones is if the agent chooses  $e_1 > e_1^*$ , causing  $S(e)$  to overlap with region II. (If  $e_1 = e_1^*$ , then  $e_p = e_p^*$ . If  $e_1 < e_1^*$ , then the principal is worse off, since his payoff would then be below the first-best one.) But, using  $f_{12}^1 \geq 0$ , the best response of



the principal would then be to choose some  $e_p \geq e_p^*$ , placing  $S(e)$  entirely in region IA and making agent 1 worse off. Q.E.D.

The main reason for rewarding agent 1 (2) in region II (III) is now clear. This serves to disrupt collusive agreement among the agents. The constancy of wages over regions I, II, and III and complementarity of inputs disrupts agent-principal coalitions.

An informational source of economies of scale is revealed by this analysis. Suppose agents have the same production and utility functions for the sake of discussion. If  $C(e_p, 2) = 2C(e_p, 1)$ , then no technological economies of scale are present. Nevertheless, one principal employing two agents would be more profitable than two pairs of a principal and one agent (holding output prices constant across the regimes). The former organizational form can resolve the incentive problems and ensure efficient production while the latter cannot. If  $C(e_p, 2) > 2C(e_p, 1)$ , i.e. if technological diseconomies of scale exist, then a trade off characterizes formation of the larger organization. The larger organization might be more efficient. The implications for market structure are clear. Note, too, that this argument for informational economies of scale does not generally apply to the standard (moral-hazard) explanation for employment of multiple agents. The correlation in random influences on agents' outputs that underlies the standard model is usually described as being independent of the organizational form, e.g. deriving from "market conditions." Agents' rewards can then be inter-linked whether or not multiple principals are compensating the agents.<sup>13</sup>

### 3.5 Conclusions

Double moral hazard induces a second-best contract in the standard principal-agent relationship. The information contained in one output does not allow a contract to distinguish agent shirking from principal shirking. Employment of two agents that share the principal's input and the consequent observation of two outputs permits stochastic identification of a shirker. Low output of just one agent is indicative of her shirking. Dually low outputs, on the other hand, are indicative of principal shirking. In the model of this paper, double moral hazard can be resolved perfectly. Unilateral shirking by an agent generates conclusive evidence of such with positive probability. This permits harsh punishment without risk of error. The presumption of moving supports on outputs underlies the use of such a forcing contract. The next essay pursues the natural extension to non-moving supports on outputs. It appends double moral hazard to Holmstrom's model. Although the first-best is not attainable, the optimal contract inter-links agents' rewards whether or not correlation in random effects is present. Further, gains from adding more agents result, independent of technological considerations.

The broad outlines of a common compensation structure are not too different from the contract I have described: early promotion for superior performance; job loss for serious laggards; and, otherwise, fairly constant salaries until promotion (e.g. cost of living raises). But this may derive instead from the organization's response to the combination of moral hazard and correlated randomness or self-selection constraints.<sup>14</sup> Testing the validity of the various explanations presents an interesting empirical challenge.

Notes

1. It may appear that the failure to attain a first-best outcome with only one agent derives from technological considerations since our model presumes the input is public to two agents. Depending on the model's specification, especially the principal's cost function, technological gains from employing two agents may or may not be present. We show that gains exist from employing two agents whether or not technological gains are present, because incentive problems are resolved.
2. Demski and Sappington (1984) show that similar contractual gains arise in multiple-agent, hidden-information environments. See also Ma, Moore, and Turnbull (1988).
3. Alternative interpretations of the above example of test construction illustrate the distinction. The example is borrowed from Nalebuff and Stiglitz who suggest that exams are curved to correct for randomness in their difficulty, while preserving student incentives to study. One model where this makes sense presumes relative performance on exams can be fully diagnostic (but for idiosyncratic randomness in student performance) independent of the absolute difficulty of the exam. It is efficient for the teacher to write the exam as quickly as possible, with its level of difficulty a random realization. Suppose instead that an exam must be of the appropriate difficulty to be diagnostic, the writing of which requires care on the part of the principal. In contrast to the previous model, a potential for principal moral hazard arises. The results of this paper imply that each student's grade should again depend on other students' performances, even absent any randomness in the model (and also that the teacher should be "graded" based on student performances).
4. Several of the aforementioned papers also analyze rank-order tournaments, but not motivated by the principal's moral hazard next discussed.
5. Rank-order reward schemes do correct the type of principal moral hazard we study. They are too crude, however, to be efficient in our information setting.
6. Itoh's model has the elements of Carmichael's described next. His predictions are then not robust to the consideration of general reward structures.
7. It is interesting that the linear reward function actually is coalition proof in the special case of a production function that is additively separable in the agent's and principal's inputs and the random term. It is not, however, an efficient reward function.
8. The presumption is just that the principal increases profits by employing a second agent. One can also consider the issue of market structure, comparing the efficiency of two principal-agent pairs to one principal employing two agents.

This is discussed below. For now, the standard (implicit) assumption is made that two separate organizations, with two principals, is infeasible.

9. Our model presumes agents are risk averse. Double moral hazard is never resolvable when one agent is employed. If the agent is risk neutral, then double moral hazard is generally nonresolvable, but can be corrected in some (very) special cases. (An example of this is available from the authors.) The contract we present below with two agents also resolves fully double moral hazard when the agents are risk neutral.
10. How the dimensions of  $S(e)$  vary with  $e$  depends on  $f_{13}^i$  and  $f_{23}^i$ . Our results apply in any case.
11. Agent shirking yields an  $S(e)$  that is fully contained in region IB for many production functions. Examples include all production functions  $f^i$  that are additively or multiplicatively separable in  $\theta_i$ . (See the previous footnote.)
12. Side payments between coalition members are precluded. The only enforceable payments are those dictated by (WF)!
13. The informational requirements for contract enforceability are consistent across the regimes: outputs of every agent must be public knowledge. One might argue that it is cheaper to verify outputs emanating from within a single organization. Explicit consideration of verification costs probably warrants study and may indicate economies of scale more consistent with the standard multi-agent analyses. Also, if common uncertainties across agents derive from the organization employing them, then, again, an explanation for economies of scale emerges. Our point is that this is not the usual argument; and one should ask if such common "randomness" is truly random, or within the principal's control.
14. See Bhattacharya and Gausch (1988) and the references therein for analysis of self-selection issues.

## CHAPTER 4

### DOUBLE MORAL HAZARD AND MULTIPLE AGENTS WITH A NONMOVING SUPPORT ON OUTPUT

#### 4.1 Introduction

In this chapter I continue to study the double moral hazard problem in a multiple agent setting. It differs from the previous chapter in that here I analyze a case where, though the effort levels influence the probability distribution of the outputs, the support, i.e., the lower and the upper bound of the distribution, is independent of them. In other words, I consider a nonmoving support on output. The main focus continues to be on the incentive issues of employing multiple agents in the presence of double moral hazard. The basic model remains the same, i.e., I consider a production process where each agent produces an observable output that depends on the agent's unobserved effort level, the principal's unobserved effort level (which is common to all agents), and an unobservable random error term. It is demonstrated that employment of multiple agents can partially resolve the double moral hazard problem. This conclusion stands in contrast to the existing literature. In particular, I distinguish the well-established incentive to employ multiple agents' outputs to monitor one another when random components are correlated from the incentive studied here to engage more workers to correct the problem of double moral hazard.

The moral hazard framework employed in this essay is analogous to that of Holmstrom (1979). I extend Holmstrom's basic hidden action model by including another agent and by allowing the principal to take an active part in the production process by supplying one factor of production. My model examines the case where the principal is risk neutral, the agents are risk averse, and all of the parties select their effort levels noncooperatively. I start by analyzing a model with one agent and one principal, and then show that an improvement on their performance is possible by employing another agent in the production process. I present some simulation results that illustrate some of the analytical findings.

The organization of the paper is as follows. Section 4.2 describes the basic model and underlying assumptions. In Section 4.3, I report the results in the case of one principal and one agent. Simulation results are presented in Section 4.4. Section 4.5 examines the multiple agent case, and, finally, Section 4.6 contains concluding remarks.

#### 4.2 Model and Assumptions

There are two production technologies involving one principal and two agents. The agents are indexed  $i = 1, 2$ . The principal's effort is a public input common to both agents. The objective of the principal is to maximize the expected net profit. Each agent's output is determined by the production function  $q_i = f^i(e_i, e_p, \theta_i)$ ,  $i = 1, 2$ , where  $q_i \in [0, \infty)$  is the  $i$ th agent's output;  $e_i \in [0, \infty)$  is effort of agent  $i$ ;  $e_p \in [0, \infty)$  is the effort of the principal; and  $\theta_i$  is a continuous random variable (representing the random state of nature) with density function  $h_i(\theta_i)$ . I assume that  $f^i(\cdot)$  is twice continuously

differentiable with positive marginal productivity of effort, and a higher  $\theta$  realization corresponding to a more productive state.

- A4.1.  $f_{e_i}^i \geq 0$  ;  $f_{e_p}^i \geq 0$  ; and  $f_{\theta_i}^i > 0$ ; for  $i = 1, 2$   
 with  $f_{e_i}^i = 0$  ;  $f_{e_p}^i = 0$  ; at  $[\theta_i = \theta_i^{\min} ; \theta_i = \theta_i^{\max}]$  and with strict inequality for some  $q_1, q_2$ .

Each agent is assumed to be risk-averse. The utility function of agent  $i$ ,  $U^i = U^i(w_i(q_1, q_2), e_i)$ , is defined over monetary compensation ( $w_i(q_1, q_2)$ ) and effort ( $e_i$ ). Note agent  $i$ 's pay-off may depend on his own output, as well as the output of agent  $j$ . As will be seen later, an interdependent wage scheme can be employed to discipline the agents as well as the principal. This stems from the fact that in the presence of the common input supplied by the principal, performance of one agent provides information not only about the performance of the other agent but also that of the principal. I further assume that each utility function is additively separable in the monetary compensation and the effort. Thus,

- A4.2.  $U^i = U^i[w_i(q_i, q_j)] - d^i(e_i)$

where  $U_{w_i}^i > 0$ ,  $U_{w_i w_i}^i < 0$ ,  $d_{e_i}^i > 0$ , and  $d_{e_i e_i}^i > 0$ .

A4.2 states that the agent's utility is increasing in monetary compensation at a decreasing rate and that his disutility rises at an increasing rate as he works harder. The principal is assumed to be risk-neutral, and the residual claimant on output. Accordingly, her utility function is represented by  $\sum_{i=1}^2 q_i - \sum_{i=1}^2 w_i - c(e_p)^1$ , where  $c(e_p)$  is her cost of supplying effort. This cost is increasing in  $e_p$  at an increasing rate.

The information structure in the problem is as follows. The final output produced by each agent is publicly observed, but the efforts supplied (simultaneously) are observed only by the respective supplier. The distributions of  $\theta_i$  are common knowledge, but the actual realizations of  $\theta_i$  are not observable. Compensation schemes can then be made contingent only on outcomes  $(q_1, q_2)$ .

The timing in the model is as follows. First, the principal offers the contracts which the agents can accept or reject. If either rejects, the game ends. Otherwise both the principal and the agents then take their respective actions simultaneously, i.e., supply the Nash equilibrium efforts. The outputs are then observed, and payments are made as promised.

In order to solve my problem, I follow the approach of Mirrlees and Holmstrom, and suppress the  $\theta_i$ 's and treat the  $q_i$ 's as random variables with a joint probability distribution  $G(q_1, q_2 | e_1, e_2, e_p)$ .<sup>2</sup> Given the distributions of  $\theta_i$ ,  $G(q_1, q_2 | e_1, e_2, e_p)$  characterizes the joint distribution of  $q_i$ 's, which can be deduced by using the relationship  $q_i = f(e_i, e_p, \theta_i)$ . The stochastic counterpart of the positive marginal product of inputs in A4.1 is the first-order stochastic dominance, which can be presented as follows:

A4.3.  $G_{e_1}(q_1, q_2 | e_1, e_2, e_p) \leq 0$  and  $G_{e_p}(q_1, q_2 | e_1, e_2, e_p) \leq 0 \quad \forall q_1, q_2, e_1, \text{ and } e_p$ , with strict inequality for some  $q_1, q_2$ .

A4.3 indicates that any increase in the effort level of the agent or the principal would have a nonnegligible effect on the joint distribution of  $(q_1, q_2)$ , and shift it to the right in the sense of first-order stochastic dominance. Intuitively it means that higher efforts lead to higher outputs in the sense of first-order stochastic dominance. The density function



of  $G(q_1, q_2 | e_1, e_2, e_p)$  is denoted by  $g(q_1, q_2 | e_1, e_2, e_p)$ . I assume that  $G(\cdot)$  is thrice continuously differentiable.

A4.4. The monotone likelihood ratio property holds, i.e.,  $\frac{g_{e_1}}{g}$  and  $\frac{g_{e_2}}{g}$  are increasing in  $q_i$ .

Intuitively, this assumption indicates that higher levels of output are more likely to emerge with higher level of effort.

A4.5. The first-order approach is valid.

The first-order approach involves replacing the original incentive compatibility constraints (which state that both the principal and the agent would choose the effort levels that are in their best interest) by the first-order conditions of the corresponding maximization problem. It assumes further that these first-order conditions define a unique solution.

### 4.3 Double Moral Hazard with One Agent

As a benchmark, I first address the simplified version of the problem with a single agent. The principal's problem in this case is to choose the wage schedule  $w_i(q_i)$  and the effort levels  $e_i$  and  $e_p$  to maximize her utility. Following Holmstrom (1979), this problem can be described formally as follows:

$$\text{Maximize } w_i(q_i), e_i, e_p \quad \int_0^{\infty} [q_i - w_i(q_i)] g(q_i | e_i, e_p) dq_i - c(e_p) \quad (\text{PA-1})$$

Subject to:

$$\int_0^{\infty} U^1[w_i(q_i)] g(q_i | e_i, e_p) dq_i - d^1(e_i) \geq 0 ; \quad (4.1)$$

$$\int_0^{\infty} U^1[w_i(q_i)] g_{e_i}(q_i | e_i, e_p) dq_i - d_{e_i}^1(e_i) = 0 ; \quad (4.2)$$

$$\int_0^{\infty} [q_1 - w_1(q_1)] g_{e_p}(q_1 | e_1, e_p) dq_1 - c_{e_p}(e_p) = 0 . \quad (4.3)$$

The objective function is the principal's expected utility. Constraint (4.1) is the agent's participation constraint, which guarantees the agent a minimum expected utility. This reservation level of utility is normalized to be zero. Constraints (4.2) and (4.3) are the agent's and the principal's incentive compatibility constraints, respectively. These ensure respectively that both the principal and the agent choose effort levels in their self-interest to maximize their respective expected net utilities. Generally, one party's optimal choice of input would depend on the nature of the relationship between these productive inputs, and therefore on the other party's choice of input. In particular, each would do the best she or he can anticipating what the other person might do. In other words, optimal choices are the Nash equilibrium ones. Simultaneous satisfaction of reaction functions<sup>3</sup> is a means of determining Nash equilibrium values. The slopes of the reaction functions play a crucial role in determining the nature of the equilibrium.

The agent's reaction function is determined from his incentive compatibility constraint (4.2). Likewise, the principal's reaction function is determined from her incentive compatibility constraint (4.3). Following Bulow et.al, I adopt the definition of strategic complementarity, strategic substitutability and strategic independence of the two factors of production.

**Definition** The two inputs are strategic complements, strategic substitutes, or strategically independent as the two players' reaction functions (also

known as best response functions) have positive, negative, or zero slope respectively.

Define  $e_i = j(e_p)$  as the solution to (4.2). I find that  $e_i$  and  $e_p$  are strategic complements (independent) or substitutes as  $j'(e_p) \gtrless 0$ , i.e., as

$$\int_0^{\infty} U^1[w_1(q_1)] g_{e_i e_p}(\cdot) dq_1 \gtrless 0 \quad \forall (e_i, e_p).$$

Equivalently, defining  $e_p = k(e_i)$  as the solution to (4.3),  $e_i$  and  $e_p$  are strategic complements (independent) or substitutes as  $k'(e_i) \gtrless 0$ , i.e., as

$$\int_0^{\infty} [q_1 - w_1(q_1)] g_{e_i e_p}(\cdot) dq_1 \gtrless 0 \quad \forall (e_i, e_p).$$

In other words, two inputs are defined as strategic substitutes if an increase in one player's input has a negative effect on the marginal effect of the other player's input on his own expected utility. In contrast, two inputs are strategic complements if an increase in one input enhances the marginal productivity of the other. And the two inputs are independents if one does not affect the marginal productivity of the other.

It is well known that in the case of nonmoving support on output distributions, if the effort levels are not observable ex-ante or ex-post, and thus not contractible, a first-best solution cannot be obtained (see Holmstrom (1979), there is only a one-way moral hazard on the agent's part in his work).

**Definition** A contract  $(w_1(q_1))$  is first-best, if no other contract exists that gives the principal higher expected utility and gives the agent at least his reservation level of utility.<sup>4</sup> The effort levels associated with this solution are defined

as the efficient ones. Any other effort level will be referred to as sub-efficient.

A first-best solution entails Pareto optimal risk sharing that provides proper incentives to both parties. In a case of hidden actions, the observable final outcome is utilized to determine pay-offs and one can only obtain a second-best solution, that involves a trade-off between risk sharing benefits and incentives. Here I will examine the properties of the relevant second-best optimal sharing rule. Note that if the principal's choice is observable and precedes that of the agent<sup>5</sup>, then her (the principal's) choice can be written into the contract. Consequently, the problem reduces to a case of one-way moral hazard on the agent's part and has an analogous solution to Holmstrom's. Accordingly, the associated optimum risk sharing rule is characterized by:

$$\frac{1}{U_{w_1}^1[w_1(q_1)]} = \lambda + \mu \frac{g_{q_1}}{g}, \quad (4.4)$$

where  $\lambda$  and  $\mu$  are the Lagrangian multipliers on constraints (4.1) and (4.2) respectively. As shown by Borch (1962),  $w_1(q_1)$  would be Pareto optimal from a risk-sharing point of view only if the right-hand side of (4.4) is a constant. Holmstrom (1979) shows that perfect risk sharing is possible only if  $\mu = 0$  (since  $g_{q_1}/g$  is not a constant). Moreover, he shows that with first-order stochastic dominance and disutility of effort,  $\mu$  is positive. Therefore, in this case, perfect risk sharing does not characterize the optimal contract. The contract links the agent's reward to the output realization to provide incentives, at the cost of subjecting the agent to risk. Holmstrom shows the agent's effort level is sub-

efficient, in particular, the principal would like to be able to motivate the agent to exert more effort.

Now, allowing the principal's choice of effort to be unobservable as well by reintroducing (4.3) as a constraint, and letting  $\eta$  denote the corresponding multiplier, pointwise maximization of the associated Lagrangean function yields the following second-best risk sharing rule:

$$\frac{1}{U_{w_1}^1[w_1(q_1)]} = \frac{\lambda + \mu \frac{g_{c_1}}{g}}{1 + \eta \frac{g_{c_1}}{g}} \quad (4.5)$$

Noting that  $g_{c_1}/g$  is not a constant, expression (4.5) makes clear that the lack of observability of the principal's effort level leads to deviation from the Pareto optimal risk sharing between the two parties. Examination of expressions (4.4) and (4.5) indicates that the second-best outcome in the presence of double moral hazard differs from that in the case of the one way moral hazard, and involves further complexity.

I now proceed to obtain a characterization of the above second-best risk sharing rule. To demonstrate the result, first I provide the following Lemma.

**Lemma 4.1** Both the numerator and the denominator on the right hand side of expression (4.5) is positive for all  $q_1$ .

**Proof** I first show that the denominator of (4.5) is positive by examining a slightly different version of the present contracting problem. Clearly, one is strictly better off when the pay-off is higher for all  $q$  realizations. Suppose the principal

considers throwing away the proportion  $s(q)$  of the total return. Then, the principal's objective function is:

$$\begin{aligned} & \int_0^{\infty} [q_1 - w_1(q_1) - s(q_1)] g(q_1 | e_1, e_p) dq_1 - c(e_p) \\ \text{where } & \int_0^{\infty} s(q_1) g(q_1 | e_1, e_p) dq_1 \geq 0. \end{aligned} \quad (4.3.1)$$

[PA-1] includes a fourth constraint now. As a result, the new Lagrangean is:

$$\begin{aligned} L' = & \int_0^{\infty} [q_1 - w_1(q_1) - s(q_1) + \Omega(q_1) s(q_1)] g(q_1 | e_1, e_p) dq_1 - c(e_p) \\ & + \lambda \left[ \int_0^{\infty} U^1[w_1(q_1)] g(q_1 | e_1, e_p) dq_1 - d^1(e_1) \right] + \mu \left[ \int_0^{\infty} U^1[w_1(q_1)] g_{e_1}(q_1 | e_1, e_p) dq_1 - d_{e_1}^1(e_1) \right] \\ & + \eta \left[ \int_0^{\infty} [q_1 - w_1(q_1) - s(q_1)] g_{e_p}(q_1 | e_1, e_p) dq_1 - c_{e_p}(e_p) \right] \end{aligned} \quad (4.6)$$

where  $\Omega(q_1)$  is the Lagrange multiplier on the added constraint.

Pointwise maximization of (4.6) provides

$$\begin{aligned} \frac{\partial L'}{\partial s(q_1)} &= [-g - \eta g_{e_p} + \Omega(q_1)g] = 0 \quad \forall q_1 \\ \therefore \Omega(q_1) &= 1 + \eta \frac{g_{e_p}}{g} \text{ at an interior solution.} \end{aligned}$$

Using the envelope theorem, from expressions (4.3.1) and (4.6), it is immediate that  $\Omega(q_1)$ , the Lagrange multiplier associated with it is positive for all  $q_1$ . Therefore, the left hand side of the above expression is positive, indicating that the right hand side of the same is positive too. This in turn validates that the denominator of the right hand side of (4.5) is positive.

Now by A4.2, expression 4.5 > 0.

Therefore the numerator of the right hand side of 4.5 is also positive. Q.E.D.

**Proposition 4.1**

If assumptions A4.3 to A4.5 hold and if the principal's effort ( $e_p$ ) and the agent's effort ( $e_1$ ) are strategic complements or strategically

independent, then either: (i) both  $\eta$  and  $\mu$  are positive; or (ii) both  $\eta$  and  $\mu$  are negative.

Proof The proof<sup>6</sup> proceeds in the following manner. Part ia) shows that given  $\eta$  positive,  $\mu$  has to be positive. In part ib), I prove that if  $\mu$  is positive,  $\eta$  is positive too. Therefore, I conclude that in this particular case, the multipliers cannot be of opposite signs.

ia) If  $\eta > 0$ , then  $\mu > 0$ .

Suppose  $\eta > 0$ ,

Let  $w_1^\eta(q_1)$  satisfy

$$\frac{1}{U_{w_1}^\eta[w_1^\eta(q_1)]} = \frac{\lambda}{1 + \eta \frac{g_{e_r}}{g}} \quad \forall q_1.$$

Suppose  $\mu \leq 0$ , and let  $w_1^*(q_1)$  denote the optimal wage function. Then, using Lemma 1,

$$\frac{1}{U_{w_1}^\eta[w_1^\eta(q_1)]} = \frac{\lambda}{1 + \eta \frac{g_{e_r}}{g}} \geq (\leq) \frac{\lambda + \mu \frac{g_{e_r}}{g}}{1 + \eta \frac{g_{e_r}}{g}} = \frac{1}{U_{w_1}^*[w_1^*(q_1)]} \quad \forall q_1 \text{ as } g_{e_r} \geq (\leq) 0.$$

Since  $\frac{1}{U_{w_1}^\eta[w_1^\eta(q_1)]}$  is increasing in  $w_1$  for fixed  $q_1$ ,  $w_1^*(q_1) \leq (\geq) w_1^\eta(q_1)$  as  $g_{e_r} \geq (\leq) 0$ .

Note also that  $\frac{1}{U_{w_1}^\eta[w_1^\eta(q_1)]}$  is decreasing in  $q_1$  (follows from the assumption that  $\eta > 0$ ,

definition of  $w_1^\eta(q_1)$ , and A.4.4), implying that  $w_1^\eta(q_1)$  is a decreasing function. This together with A4.3 yields the following:

$$\int_0^\infty [q_1 - w_1^*(q_1)] g_{e_r}(q_1 | e_r, e_p) dq_1 \geq \int_0^\infty [q_1 - w_1^\eta(q_1)] g_{e_r}(q_1 | e_r, e_p) dq_1 > 0. \quad (4.6.1)$$

The later part of the above inequality can be deduced as follows. Integrating the second integral by parts yields:

$$[q_1 - w_1^\eta(q_1)] \int_0^\infty g_{e_1}(q_1|e_1, e_p) dq_1 - \int_0^\infty [1 - w_1^{\eta'}(q_1)] G_{e_1}(q_1|e_1, e_p) dq_1.$$

The first term of the above expression equals zero. By A4.3 and decreasing  $w_1^\eta(q_1)$ , the entire expression is positive.

From the first order conditions of (PA-1),

$$\begin{aligned} & \int_0^\infty [q_1 - w_1^*(q_1)] g_{e_1}(q_1|e_1, e_p) dq_1 + \eta \int_0^\infty [q_1 - w_1^*(q_1)] g_{e_1 e_p}(q_1|e_1, e_p) dq_1 \\ &= -\mu \left[ \int_0^\infty U^1[w_1^*(q_1)] g_{e_1 e_1}(q_1|e_1, e_p) dq_1 - d_{e_1 e_1}(e_1) \right] \end{aligned} \quad (4.7)$$

Using (4.6.1), Proposition 4.1's presumption that  $e_1$  and  $e_p$  are strategic complements or strategically independent is sufficient for the positivity of the left hand side of (3.7). With the assumption that  $\mu \leq 0$ , the right hand side is non-positive [the expression in the bracket is negative since it is the second-order condition for the agent's maximization problem which is assumed to be strictly concave]. Hence, there is a contradiction, and I conclude that  $\mu > 0$ .

ib) If  $\mu > 0$ ,  $\eta > 0$ .

This part of the proof follows the analogous procedure.

Suppose  $\mu > 0$ .

Define  $w_1^\mu(q_1)$  s.t.  $\frac{1}{U_{w_1}^\mu[w_1^\mu(q_1)]} = \lambda + \mu \frac{g_{e_1}}{g}$

Assume  $\eta \leq 0$ . Then, using Lemma 1, I get:

$$\frac{1}{U_{w_1}^\mu[w_1^\mu(q_1)]} = \lambda + \mu \frac{g_{e_1}}{g} \leq (\geq) \frac{\lambda + \mu \frac{g_{e_1}}{g}}{1 + \eta \frac{g_{e_1}}{g}} = \frac{1}{U_{w_1}^1[w_1^*(q_1)]} \quad \forall q_1, \text{ as } g_{e_1} \geq (\leq) 0.$$



Again, since  $\frac{1}{U_{w_1^\mu}[w_1^\mu(q_1)]}$  is increasing in  $w_1$ , for fixed  $q_1$ ,  $w_1^*(q_1) \geq (\leq) w_1^\mu(q_1)$  as  $g_{e_p} \geq (\leq) 0$ .

With  $\mu > 0$  and A4.4,  $\frac{1}{U_{w_1^\mu}[w_1^\mu(q_1)]}$  is increasing in  $q_1$  yielding  $w_1^\mu(q_1)$  is increasing in  $q_1$ .

This together with A4.3 to A4.5 implies the following:

$$\int_0^\infty U^1(w^*(q_1)) g_{e_p}(q_1 | e_1, e_p) dq_1 \geq \int_0^\infty U^1(w^\mu(q_1)) g_{e_p}(q_1 | e_1, e_p) dq_1 > 0$$

The strict inequality can be derived by integrating the second integral by parts as before in part ia).

From the first-order conditions of (PA-1),

$$\begin{aligned} & \lambda \int_0^\infty U^1(w^*(q_1)) g_{e_p}(q_1 | e_1, e_p) dq_1 + \mu \int_0^\infty U^1(w^\mu(q_1)) g_{e_p}(q_1 | e_1, e_p) dq_1 \\ & = -\eta \left[ \int (q_1 - w^*(q_1)) g_{e_p}(q_1 | e_1, e_p) dq_1 - c_{e_p} e_p(e_p) \right] \end{aligned} \quad (4.8)$$

Again, the left-hand side of (4.8) is positive when  $e_1$  and  $e_p$  are strategic complements or strategically independent. With the assumption that  $\eta \leq 0$ , the right hand side is negative.

Therefore, a contradiction results, and I conclude that  $\eta > 0$ .

Q.E.D.

According to Proposition 4.1, when the effort levels are either strategic complements or are independent, the multipliers can be either both positive or both negative, i.e., starting from an initial equilibrium, it is in the principal's and the agent's joint interest to have both either increase or decrease their efforts locally.

The possibility of negative multipliers is indeed intriguing. Note that the second-best solution crucially depends on the distribution of  $q_1$ , and its functional relation to  $e_1$  and  $e_p$ . This is because the outcome  $q_1$  is used as a signal about the actions  $e_1$  and  $e_p$ .

which are not directly observed. Now, in the presence of double moral hazard, higher output need not always signal higher effort levels despite stochastic dominance and monotone likelihood ratio property. Since the shape of the optimal wage scheme is determined by the information content in the observed level of output, negative multipliers can derive from effort increase resulting in increased uncertainty.<sup>7</sup> Note that in the standard one way moral hazard (on the agent's part), the above could never be an equilibrium property. The wage function will be nonincreasing in output if the multiplier is negative and consequently the agent's incentive to work would be reduced at the correct level.

Consider the simplest case with strategically independent inputs. The multipliers cannot have opposite signs since the wage function can be adjusted to correct this phenomena. For example, suppose that the principal's multiplier is positive and the agent's is negative. Clearly, this can be easily fixed by transferring some money to the agent from the principal.

Now examine the case where the inputs are strategic complements, i.e., increase in one enhances the effect of the other one. Given the second-best risk sharing rule, the principal would like the agent to put in more effort, as that would have a positive impact on the principal's net utility (evident from the first term in expression (4.7)). Now in order to provide the agent with expanded incentive to work hard, the principal herself would commit to a higher level of effort. As a result, both the multipliers would be positive. On the other hand, if due to increased uncertainty, and therefore higher

insurance payment, the principal prefers lower effort on the agent's part, the efforts being strategic complements, she would lower her effort level as a self-interested action.

When these inputs are strategic substitutes, increase in one diminishes the effect of the other one. In that case, given the second-best risk sharing rule, the Lagrange multipliers on the agent's and the principal's incentive compatibility constraints can be either of the same signs or of the opposite signs, resulting into four distinct possibilities. A close examination of the first order conditions (4.7) and (4.8) will give some insights into this case. From (4.7), an increase in the agent's input affects the principal's welfare directly by increasing her utility and indirectly (through her incentive compatibility constraint) by making the principal choose a new effort level in response to the agent's action. When the inputs are strategic substitutes, due to the negative externality of the increased effort level of the agent on the principal's effort, the principal's optimal response to an increased effort on the agent's part would be to reduce her own effort. It is possible for either of these effects (direct and indirect) to dominate the other. Clearly, there are some offsetting effects, and hence the ambiguity. Undoubtedly the intuition behind these findings is very complicated and sometimes "untractable".

The negativity of both multipliers, in the case where the two inputs are strategic substitutes, or independents, can be ruled out with the sufficient condition that  $g_{e_1}$  and  $g_{e_2}$  are of the same sign and magnitude everywhere for all  $q_1$ . The proof is provided in appendix C. No doubt that the above is a rather strict condition. Nevertheless, it illustrates the improbability of observing negative multipliers with regard to some very specific cases. Note also that the results hold even for a slightly relaxed set of

distribution functions, where  $g_{e_i}$  and  $g_{e_p}$  don't have to be of the same magnitude everywhere. The distribution function considered in the following section provides a suitable example.

#### 4.4 Simulation

This section reports results of a numerical simulation that extends the above analytical results. Suppose the risk-averse agent has a utility function given by:  $U(w_i(q_i)) = 2(w_i(q_i))^{1/2}$ . Also suppose the probability density (on output  $q_i$ ) is:  $g(q_i|e_i, e_p) = \frac{1}{e_i e_p} + 2(1 - \frac{1}{e_i e_p})q_i$  where  $e_i, e_p \in [1, \infty]$  and  $q_i \in [0, 1]$ . The corresponding likelihood ratio  $\frac{g_{e_i}}{g}$  is given by  $\frac{2q_i - 1}{e_i[2(e_i e_p - 1)q_i + 1]}$  where  $i = 1, p$ . Note here that the distribution function satisfies both first-order stochastic dominance and monotone likelihood ratio property.

One rather significant aspect of this example is that, here instead of normalizing the price to 1, I allow the price to vary. Accordingly, the first and the second-best solutions are obtained for different levels of prices, and the numerical results are presented in Table 4.1.

Before discussing the properties of the numerical solutions, let me first present some analytical results specific to this example.<sup>8</sup> From (4.5), the second-best

$$\text{optimal share is: } w_i(q_i) = \frac{[\lambda + \mu(\frac{2q_i - 1}{e_i[2(e_i e_p - 1)q_i + 1]})]^2}{[1 + \eta(\frac{2q_i - 1}{e_p[2(e_i e_p - 1)q_i + 1]})]^2} \quad (4.5.1)$$

where

$$\lambda = \frac{e_1 e_p}{3e_p - e_1^2};$$

$$\eta = \frac{e_p (e_1^2 - e_p)}{3e_p - e_1^2};$$

$$\mu = \frac{e_p (2e_p - e_1^2)}{3e_p - e_1^2}.$$

The relevant first-best problem yields:

$$w_{1\lambda}(q_1) = \lambda^2; \quad e_1 = 2\lambda; \quad e_p = (e_1^2/2); \quad \lambda = (P/48)^{1/5}.$$

Results presented in Table 4.1 reveal that in all of these cases, the second-best profit is lower than the first-best level, and as expected, the difference between these two profit levels widens with higher prices.<sup>9</sup> The second-best effort level of the agent is below his first-best effort choice for all prices. In contrast, the principal's effort choice at the solution to the second-best problem exceeds the first-best effort level. The fact that

TABLE 4.1  
SIMULATION RESULTS

P	FIRST-BEST			SECOND-BEST					
	profit	$e_1$	$e_p$	profit	$e_1$	$e_p$	$\lambda$	$\mu$	$\eta$
48	27	2	2	26.69	1.65	2.15	0.95	0.91	0.33
72	42.15	2.2	2.3	41.7	1.8	2.5	1.05	1.03	0.43
96	57.40	2.3	2.7	56.88	1.9	2.7	0.93	0.88	0.45
120	72.79	2.4	2.9	72.16	2.1	3.2	1.29	1.23	0.75

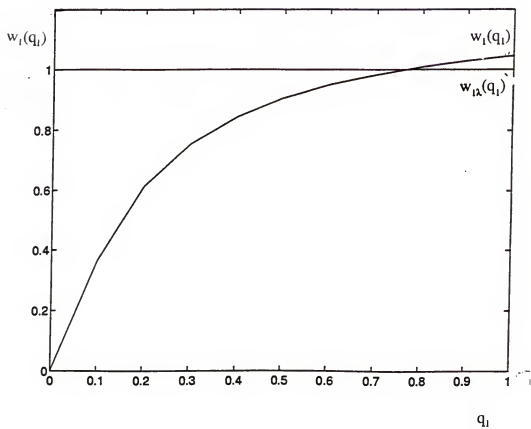


Figure 4.1

THE SECOND-BEST SOLUTION FOR THE EXAMPLE WITH  $P = 48$

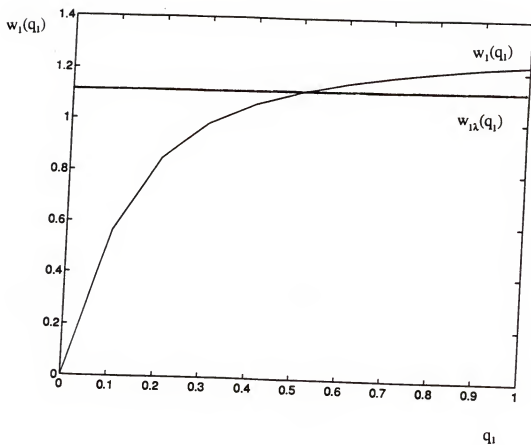


Figure 4.2

THE SECOND-BEST SOLUTION FOR THE EXAMPLE WITH  $P = 72$

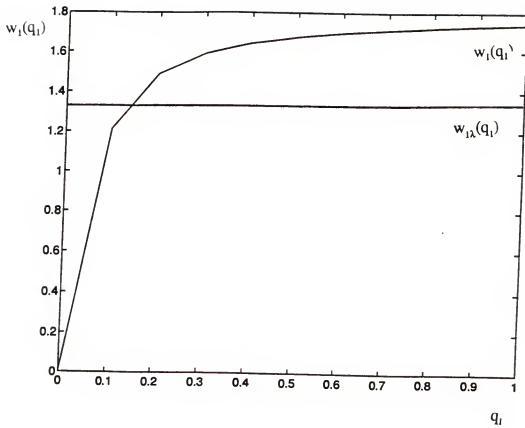


Figure 4.3

THE SECOND-BEST SOLUTION FOR THE EXAMPLE WITH  $P = 96$



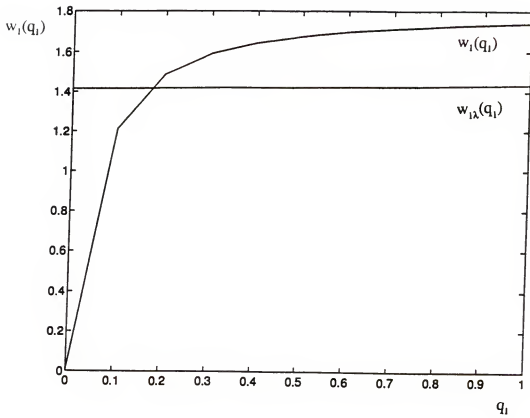


Figure 4.4

THE SECOND-BEST SOLUTION FOR THE EXAMPLE WITH  $P = 120$

the two inputs are strategic substitutes for all these prices explains this fact. Recall that, by definition, two inputs are strategic substitutes when a reduction in one reinforces the marginal effect of the other one. Therefore, when the agent reduces his effort, it makes the principal's effort more productive, and accordingly her effort will be increased.

One appealing result of this example is that, in all of the four cases, both the multipliers are positive, indicating that it is in both parties joint interest to increase the effort levels locally. This suggests that, though in the general model I was unable to rule out the possibility of both multipliers being negative, that is unlikely to arise normally.

Next, let me examine the nature of the wage schemes (see figures 4.1 - 4.5). In all these cases the wage schedule is concave, and the reward is increasing with output at a decreasing rate. The output threshold below which the second-best wage falls short of the first-best wage varies inversely with price. Moreover, from the shape of the reward functions, it is apparent that the reward varies rapidly for sufficiently low level of outputs.

#### 4.5 Double Moral Hazard with Multiple Agents

In this section I extend the problem addressed in the previous section by introducing another risk-averse agent whose productive input ( $e_2$ ) along with the public input supplied by the principal produces  $q_2$  units of the same product. This output also depends on an exogenous random state of nature experienced by the second agent. This environmental shock, however, may or may not be related to the randomness faced by agent one. Accordingly, here I consider two distinct cases: i) that where the two error

terms are stochastically independent; and ii) that where the two error terms are correlated. Existing literature has shown that in the presence of common environmental shocks, relative performance evaluation provides information about each agent's productivity and therefore is a useful monitoring device (see Holmstrom (1982); Green and Stokey (1981); Lazear and Rosen (1981); Ma (1988); Mookherjee (1984); and Nalebuff and Stiglitz (1983)). The main contribution of this essay is to show that, even in the absence of correlation in the random components of the two production functions, the additional agent's output can be used to evaluate the efficiency of the production process in the case where the principal supplies a common input. In such a situation, one output is informative about the other one to the extent that they are sharing a common public input. Recall the example used in the previous chapter for clarification. If the fast food franchisor does not spend optimal effort at quality control across the entire system, or at national advertising, it is likely to observe similar low output for all franchisees. Whereas, in the event where one of the franchisees is shirking, it is more likely to observe a low output only for that firm. It is clear that the level of both outputs provide information about the principal's as well as that of the agents' productivities and thus helps identifying the shirker.<sup>10</sup> Therefore, better incentives can be provided by making wage payments to each agent contingent on the output of the other one.

With one principal and two agents, the principal's problem [PA-2], is defined as follows:

$$\text{Maximize}_{w_i(q_1, q_2), e_1, e_p} \int_0^\infty \int_0^\infty \left[ \sum_{i=1}^2 q_i - \sum_{i=1}^2 w_i(q_1, q_2) \right] g(q_1, q_2 | e_1, e_2, e_p) dq_1 dq_2 - C(e_p)$$

subject to:

$$\int_0^\infty \int_0^\infty U^i[w_i(q_1, q_2)] g(q_1, q_2 | e_1, e_2, e_p) dq_1 dq_2 - d^i(e_i) \geq 0 \quad \text{for } i = 1, 2 \quad (4.9)$$

$$\int_0^\infty \int_0^\infty U^i[w_i(q_1, q_2)] g_{e_i}(q_1, q_2 | e_1, e_2, e_p) dq_1 dq_2 - d_{e_i}^i(e_i) = 0 \quad \text{for } i = 1, 2. \quad (4.10)$$

$$\int_0^\infty \int_0^\infty \left[ \sum_{i=1}^2 q_i - \sum_{i=1}^2 w_i \right] g_{e_p}(q_1, q_2 | e_1, e_2, e_p) dq_1 dq_2 - C_{e_p}(e_p) = 0 \quad (4.11)$$

Point-wise maximization of [PA-2] yields the following second-best risk sharing rule:

$$\frac{1}{U_{w_i}^i[w_i(q_1, q_2)]} = \frac{\lambda_i + \mu_i \frac{g_{e_i}}{g}}{1 + \eta \frac{g_{e_i}}{g}} \quad (4.12)$$

where  $\eta$  is the Lagrangian multiplier on (4.11), and  $\lambda_i$  and  $\mu_i$  are the Lagrangian multipliers on constraints (4.9) and (4.10) for  $i = 1, 2$ .

First consider the case where the random error terms  $\theta_1$  and  $\theta_2$  are stochastically independent. In this case,

$$g(q_1, q_2 | e_1, e_2, e_p) = g_1(q_1 | e_1, e_p) g_2(q_2 | e_2, e_p). \quad (4.12.1)$$

Consequently, the second-best risk sharing rule reduces to:

$$\frac{1}{U_{w_i}^i[w_i(q_1, q_2)]} = \frac{\lambda_i + \mu_i \frac{g_i e_i}{g}}{1 + \eta \left[ \sum_{i=1}^2 \frac{g_i e_i}{g_i} \right]} \quad i = 1, 2 \quad (4.13)$$

**Proposition 4.2**

Even when  $\theta_1$  and  $\theta_2$  are independent, each workers' compensation optimally varies with the other's output in the solution to [PA-2].

Proof Differentiating (4.13) with respect to  $q_j$ ,  $i \neq j$ , I get,

$$\frac{-U_{w_i w_i} \frac{\partial w_i}{\partial q_j}}{[U_{w_i}(\cdot)]^2} = \frac{-(\lambda_i + \mu_i \frac{g_i e_i}{g_i}) \eta \frac{\partial}{\partial q_j} [\frac{g_j e_j}{g_j}]}{[1 + \eta (\sum_{i=1}^2 \frac{g_i e_i}{g_i})]^2} \quad (4.14)$$

First let me show that

$$\lambda_i + \mu_i \frac{g_i e_i}{g_i} \neq 0$$

$$\text{Suppose } \lambda_i + \mu_i \frac{g_i e_i}{g_i} = 0 \quad (4.15)$$

For the above to be true, one of the following has to be correct.

Either i) both  $\lambda_i$  and  $\mu_i = 0$ , or ii)

$$\lambda_i = -\mu_i \frac{g_i e_i}{g_i}$$

If  $\mu_i = 0$ , then the agent would pick an effort level in violation to the incentive compatibility constraint,  $\therefore \mu_i \neq 0$ . Using envelope theorem, from the Lagrangean of [PA-2], it can be shown that  $\lambda_i > 0$  (Note also that  $\lambda_i = 0$  implies that the payment to agent is  $-\infty$ ). Therefore i) is ruled out.

Now the fact that  $\lambda_i, \mu \neq 0$ , and A4.4, ii) can hold at most at one point.

$\neq$  (4.15) cannot be true in general.

By A4.2, A4.4, (4.12.1), and the fact that the multipliers do not vanish, the R.H.S.

of expression (4.14)  $\neq 0$ .

Therefore, by A4.2,  $\frac{\partial w_i}{\partial q_j} \neq 0$

Q.E.D.

Proposition 4.2 shows that despite the independence between  $\theta_1$  and  $\theta_2$ , in the presence of double moral hazard, agent  $i$  ( $j$ ) is rewarded based not only on his own output but also on agent  $j$ 's ( $i$ 's) output. In other words, there exists an incentive to connect the wages across the agents. The reason for choosing an interdependent wage scheme here is different from that in the literature on relative performance evaluation. Here, because of the presence of the common input supplied by the principal, performance of one agent provides information about the effort of the principal as well as that of the other agent. In particular, agent 2's output  $q_2$  becomes informative about agent one's effort level  $e_1$  in general since when  $q_2$  is large, there is a certain probability that  $e_p$  is large, and  $e_p$  being the public input, the likelihood of observing a low  $q_1$  is higher when agent 1 shirks. Therefore, agent one will be penalized if  $q_1$  and  $q_2$  differ significantly.

Note that the second agent not only monitors the principal's performance here, but also prevents the first agent from shirking. This can be verified immediately by examining expression (4.14). It is clear from the denominator of the same expression that the true value of  $e_p$  is signalled from both outputs. Therefore shirking by the principal is likely to result in stochastically low output in both cases, and can be more easily identified. Naturally the principal's action (and thus the first agent's performance) in this case can be monitored by hiring a second agent. Notice also that at the solution of the second-best problem, agent one's pay-off depends on the marginal distribution of agent two's output as well (captured in the term  $\frac{g_{2e_p}}{g_2}$ ). Accordingly, both agents' output should be taken into consideration while providing incentive to an individual agent.<sup>11</sup>

Next I consider the case where  $\theta^i$  and  $\theta^j$  are correlated. In this case, agent  $i$ 's ( $j$ 's) wage function still depends on agent  $j$ 's ( $i$ 's) output. However, the wage scheme for one worker is not necessarily decreasing in the output of the other worker any more. Instead, now there is a trade-off and the wage can be either increasing or decreasing in the other party's output. A close examination of the following expression can provide me with some insight into it.

$$\frac{-U_{w_i w_i}^i \frac{\partial w_i}{\partial q_j}}{[U_{w_i}^i]^2} = \frac{(1 + \eta \frac{g_{e_i}}{g}) \mu_i \frac{\partial}{\partial q_j} [\frac{g_{e_i}}{g}] - (\lambda_i + \mu_i \frac{g_{ie_i}}{g}) \eta \frac{\partial}{\partial q_j} [\frac{g_{e_i}}{g}]}{[1 + \eta \frac{g_{e_i}}{g}]^2} \neq 0 \quad (4.16)$$

As opposed to the case where the random error terms are uncorrelated, I can now identify two distinct effects characterizing the incentive to base the payment on relative performance. The first one is the well-known incentive to link the agents' wages when they face some common uncertainty. The second one arises due to the presence of double moral hazard where the public input is supplied by the principal. Notice here that because of the presence of common uncertainty, I am employing the joint distribution of  $q_1$  and  $q_2$ . Since the knowledge of all marginal densities does not in general imply knowledge of the joint density and vice versa, an observed high value of  $q_2$  does not always signal a low value of  $e_1$ . In particular, an increase in  $q_2$  due to an increase in the effort level of agent 2 might shift the joint distribution in such a way that it might signal a high  $e_1$  as well. In that case,  $\frac{\partial}{\partial q_j} [\frac{g_{e_i}}{g}] > 0$ . In the case of strategically complementary inputs, the two terms in the numerator of expression (4.16) are of opposite signs. Now depending on the relative strength of them, the wage of agent one can be increasing or

decreasing in the other agent's output, i.e., there is a possibility that one agent's compensation is increasing in the other agent's output. On the other hand, a more usual case is where a significantly higher value of  $q_2$  signals a low  $e_1$ . Accordingly,  $\frac{\partial}{\partial q_j} [\frac{g_{e_i}}{g}] < 0$ , and in that case, the effect of double moral hazard and the correlation in the error terms work in the same direction. As a result, agent one is penalized when  $q_2$  is significantly higher than  $q_1$ .

Proposition 4.3      Irrespective of the stochastic dependence or independence between  $\theta^i$  and  $\theta^j$ , the principal is strictly better-off with two agents.

Proof With two agents, the principal clearly has the option of rewarding the agents based solely on his output as before in [PA-1]. However, the principal is in fact choosing a different payment scheme that is dependent on both outputs  $q_i$  and  $q_j$ . Hence she must be strictly better-off with the second mechanism design. Q.E.D.

Note that the above result holds independent of technological economies of employing more agents. To explain it further, let me assume that the agents are symmetrical, i.e., they have the same utility functions, production functions, and effort levels. Now if  $C(e_p, 2) = 2C(e_p, 1)$ , then there is no technological economies of scale. Regardless, an organization with one principal employing two agents will be more lucrative than two separate organizations with one principal and one agent each. This is because in the first case, better incentives can be provided by interlinking the two agents'



wages. The result can be easily extended for  $n$  agents, and it can be shown that the solution improves with each extra agent.

#### 4.6 Conclusions

I have examined the incentive issues in a double moral hazard multiple agent setting in this paper. The main finding is that even in the absence of common uncertainties, the problem of double moral hazard can be partially resolved by employing more agents. I analyze the problem in a case of nonmoving support on outputs and show that even though the first-best is not attainable, the solution improves with two agents. I find that here monitoring is possible by connecting the two wages since the principal's input is a public input. As against this, when there is common shock in the environment, the incentive to make payments contingent on both outputs are twofold -- first is due to the common uncertainty in the system and the second is due to the presence of double moral hazard. Unfortunately, I cannot distinguish these effects completely since they are intertwined. However, in this case there is a certain (though unlikely) possibility that one agent's compensation is actually increasing in the other agent's output. In addition to this, I also see that to a large extent, the nature of the solution of the double moral hazard problem with one agent depends on the specific relationship between the factor of production. The numerical example provides some insight into this case. However, at this point I don't have many sharp predictions as to how the solution changes with multiple agents. One way of complementing that is to find a numerical solution for an

example with two agents. Presently those results could not be provided due to the complexity and difficulty of the problem.

### Notes

1. Note that the utility function of the principal is her expected profit level. The gross revenue earned is  $p_1q_1 + p_2q_2$  where  $p_1, p_2$  are the respective level of prices. Here, without loss of generality I assume that  $p_1 = p_2 = 1$ .
2. In the case where there is just one agent, the corresponding distribution and density function of  $q_1$  would be represented by  $G(q_1|e_1, e_p)$  and  $g(q_1|e_1, e_p)$  respectively.
3. Reaction functions depict what a player would do if he/she were to learn the level of his/her opponent's action. In other words, reaction functions represent the best effort choice of one party for each possible choice of the other one.
4. Note that the incentive compatibility constraints are dropped here.
5. As long as the principal's choice is observed at some point, the problem has an analogous solution outcome to that of Holmstrom's model. It is not the timing but the observability that matters. However, depending on the timing, the contract can have different features.
6. This proof is an extension of the proof of Proposition 1 in "Moral Hazard and Observability" by Holmstrom (1979).
7. For example, although hard work on either party's part is likely to increase the output and the monetary return, the agent will be subjected to a risk of income fluctuations associated with the signalled level of performance. As a result, the principal needs to compensate the agent by paying him more. If the resulting cost is higher than the benefit, lower level of efforts will be preferred. In other words, here the risk-sharing effect is so big that it outweighs the incentive effect, and as a result, a better solution will be obtained with less amount of work.
8. The optimum choice of effort levels are unique here. This can be readily verified from the fact that (4.2) and (4.3) are concave in  $e_1$  and  $e_p$  respectively for the optimal  $w_1(q_1)$  in (4.5.1).

9. Note that the second-best profit is fairly close to the first-best one in all cases. This essentially stems from the fact that the profit function here is considerably flat.
10. The party who is not putting forth the optimal level of effort is referred to as a shirker.
11. Note that with uncorrelated error terms, and  $\eta > 0$ , agent  $i$ 's ( $j$ 's) wage function is decreasing in agent  $j$ 's ( $i$ 's) output. It can be proved immediately from expression (4.14), A4.4, and the positivity of numerator and denominator of expression (4.13) (follows from Lemma 1).

By making each agents' wage decreasing in the other agent's output, the contract maintains work incentives for the principal while preserving the work incentives for the agents. When the principal works harder, and both outputs rise, the "increase" in wages is tempered. At the same time, the agents are not penalized for their increased effort. This also explains why one gets the opposite results if  $\eta < 0$ .

## CHAPTER 5

### SUMMARY AND CONCLUSIONS

This work focuses on the study of various kinds of incentive issues. Chapter 2 presents evidence consistent with collusion sometimes being found in the highway construction auction market. It is the concept of repeated interaction amongst the bidders that motivates this study and we test the impact of repeat bidding on winning bid prices. My finding that bid prices are higher when repeat bidding is more prevalent supports the hypothesis that repeat bidding is conducive to collusion. Similarly, the fall in bid prices as the number of bidders increases implies that collusion is less common when there are more bidders. Thus the number of bidders provides some information to procurement agencies. The results also suggest that collusion is normally nonexistent in these markets if there are at least eight bidders. My other findings indicate that collusion is more likely to exist in large projects and when business is slow.

One potential problem in using repeated interaction among bidders as an indicator of collusion is that the repeated interaction itself might be an endogenously determined factor. Possible factors that might influence the incidence of the same groups of firms bidding together include the geographical area of the projects, and the size and type of the project, or other similarities between projects. This can be explained by the fact that it is more likely that the same firms will bid on similar types of projects, (e.g. adding a

lane, constructing a bridge etc.). Also, it may be easier to collude if the firms are located in the same area since the participants should be few enough for effective and efficient coordination. Accordingly, one could use the number of bidders per area instead of using the number of bidders per project to measure collusion. More importantly, it would be interesting to see what actually determines the participation and the number of bidders per project in such a strategic dynamic setting. Unfortunately, the data at this time do not permit me to explore these questions.

In chapter 3 and 4, I examine the incentives to employ multiple agents in a double moral hazard setting. Chapter 3 shows that in the presence of moving support on output, the problem of double moral hazard can be fully resolved by employing multiple agents. There is gain from making one agent's payoff dependent on the other agent's output, independent of correlation in the random components that affect their outputs. Dissimilar output identify shirking by an agent, while similar, low outputs identify shirking by the principal. Moreover, in this case, a contract exists that yields the first-best efforts as the unique and coalition-proof Nash equilibrium. The coalition proofness prevents agents from colluding to exploit the contract's reading of multiple low outputs as indicative of shirking on the part of the principal.

In chapter 4, I continue to investigate the same problem in a case where there is a nonmoving support on the distribution of outputs. The main finding is that even in the absence of common uncertainties, the problem of double moral hazard can be partially resolved by employing more agents. I show that even though the first-best is not attainable, the solution improves with two agents. It is found that monitoring is possible

by making the wages contingent on both outputs since the principal's input is a public input. On the other hand, when there is common random disturbance affecting the environment, there are two distinct incentives to interlink the payment schemes. The first one is due to the common uncertainty in the system and the second one is due to the presence of double moral hazard. Unfortunately, I cannot, at this point, distinguish these effects completely since they are intertwined. However, in this case, there is a certain (though unlikely) possibility that one agent's compensation is actually increasing in the other agent's output. Moreover, it is also seen that to a large extent the nature of the solution of the double moral hazard problem with one agent depends on the specific relationship between the factors of production. In particular, at the solution to the second-best problem, starting from an initial equilibrium, both the principal and the agent would like to increase or decrease their respective efforts depending on whether the inputs are strategic substitutes or complements. Presently I don't have many sharp predictions about how the solution improves with more agents. One way of obtaining further insight into this problem is to extend the example for two agents.

## APPENDIX A

### NON-ATTAINABILITY OF FIRST-BEST OUTCOME WITH ONE AGENT

We simplify the notation. Since only one agent is present, drop the subscript/superscript on values/functions that was used to distinguish the agents (e.g. let  $e$  denote the agent's effort and  $q = f(e, e_p, \theta)$  denote the output).  $C(e_p)$  denotes the principal's effort cost and  $h(\theta)$  the density function of the random element, the latter strictly positive over  $[\theta_{\min}, \theta_{\max}]$  for simplicity. The modified first-best problem is:

$$\begin{aligned} \text{MAX}_{e, e_p, q, w(q)} \quad & \int_{\theta_{\min}}^{\theta_{\max}} [q - w(q) - C(e_p)] h(\theta) d\theta \\ \text{s.t.} \quad & \int_{\theta_{\min}}^{\theta_{\max}} U(w(q), e) h(\theta) d\theta \geq 0 \\ & q = f(e, e_p, \theta). \end{aligned} \tag{FBP-A}$$

Let a  $^{**}$ -superscript indicate solution values. They satisfy:

$$w^{**}(q) = w^{**} \quad \forall q \in [f_m, f_x]; \tag{A1}$$

$$U(w^{**}, e^{**}) = 0; \tag{A2}$$

$$E[f_1(e^{**}, e_p^{**}, \theta)] = -U_2/U_1; \quad \text{and} \tag{A3}$$

$$E[f_2(e^{**}, e_p^{**}, \theta)] = C'(e_p^{**}); \tag{A4}$$

where  $f_m = f(e^{**}, e_p^{**}, \theta_{\min})$  and  $f_x = f(e^{**}, e_p^{**}, \theta_{\max})$ . Condition (A1) indicates complete insurance is provided to the agent. Condition (A2) says that the agent obtains only her reservation utility. Conditions (A3) and (A4) are the marginal conditions describing the efficient efforts.

The second-best problem adds to (FBP-A) the incentive compatibility constraints:

$$e \in \text{ARGMAX } E[U(w(q), e)]; \quad (\text{A5})$$

and

$$e_p \in \text{ARGMAX } E[q - w(q) - C]. \quad (\text{A6})$$

**Proposition A1** No contract exists that engenders the first-best outcome in the second-best problem.

**Proof** The proof is by contradiction, so assume (A1) -(A4) are satisfied. Using (A1), to satisfy (A5),  $w(q)$  must increase discretely at  $q = f_m$ . Assuming the derivative exists, we have:

$$\frac{\partial E[U]}{\partial e} = E[U_2] + \int_{\theta_{\min}}^{\theta_{\max}} U_1 w' f_1 h(\theta) d\theta.$$

Condition (A1) implies  $w' = 0 \ \forall \ q \in (f_m, f_x)$ . Hence, because  $E[U_2] < 0$ ,  $E[U]$  can be increased by decreasing  $e$  below  $e^{**}$  unless " $w'(f_m) = \infty$ ." Then, for  $w(q)$  discontinuous at  $q = f_m$  and  $\Delta e < 0$ ,



$$\Delta E[U] \approx \left\{ E[U_2] + (U_1 f_1 h) \Big|_{\theta=0_{\min}} \cdot \frac{\Delta w}{\Delta q} \Big|_{q=f_n} \right\} \Delta e.$$

Gains to the agent from decreasing  $e$  finitely below  $e^{**}$  result unless  $\frac{\Delta w}{\Delta q} \Big|_{q=f_n}$  is positive and of sufficient magnitude.

But this contradicts (A6), i.e. the principal can gain by shirking. For  $\Delta e_p < 0$ ,

$$\begin{aligned} \Delta E[q - w(q) - C] &\approx \left\{ E[f_2] - C' - (f_2 h) \Big|_{\theta=0_{\min}} \cdot \frac{\Delta w}{\Delta q} \Big|_{f=f_n} \right\} \Delta e_p \\ &= - \left\{ (f_2 h) \Big|_{\theta=0_{\min}} \cdot \frac{\Delta w}{\Delta q} \right\} \Delta e_p, \end{aligned}$$

the second line by (A4). The latter is positive, i.e. the principal would gain by shirking. Q.E.D.

## APPENDIX B

### PROOF OF UNIQUENESS IN PROPOSITION 3.1

The method of proof is to rule out vectors other than  $e^*$  as equilibrium values. Six alternatives to  $e^*$  must be rejected:

(1)  $e_p \geq e_p^*$ ;  $e_1 \geq e_1^*$ ; and  $e_2 \geq e_2^*$ ; at least one with strict inequality.

(2a)  $e_p \geq e_p^*$ ;  $e_1 \geq e_1^*$ ; and  $e_2 < e_2^*$ .

(2b)  $e_p \geq e_p^*$ ;  $e_1 < e_1^*$ ; and  $e_2 \geq e_2^*$ .

(3)  $e_p \geq e_p^*$ ;  $e_1 < e_1^*$ ; and  $e_2 < e_2^*$ .

(4)  $e_p < e_p^*$ ;  $e_1 \geq e_1^*$ ; and  $e_2 \geq e_2^*$ .

(5a)  $e_p < e_p^*$ ;  $e_1 \geq e_1^*$ ; and  $e_2 < e_2^*$ .

(5b)  $e_p < e_p^*$ ;  $e_1 < e_1^*$ ; and  $e_2 \geq e_2^*$ .

(6)  $e_p < e_p^*$ ;  $e_1 < e_1^*$ ; and  $e_2 < e_2^*$ .

Cases 2 and 5 have subcases that reverse the behaviors of the two agents.

Case 1 If  $e_p > e_p^*$  and  $e_i \geq e_i^*$   $i=1,2$ , then  $e_j < e_j^*$  is optimal for agent  $j$ ,  $j = 1,2$  and  $j \neq i$ , since  $S(e)$  could remain in region I. Hence, for such values to be equilibrium ones, it must be that  $e_p = e_p^*$ . If  $e_p = e_p^*$  and  $e_i \geq e_i^*$ , then  $e_j = e_j^*$  is optimal for agent  $j$ . But then, symmetrically,  $e_i = e_i^*$  is optimal for agent  $i$ .

### Case 2

For  $e$  satisfying (2a),  $S(e)$  will be either: (i) entirely contained in region I; (ii) entirely contained in region II; or (iii) overlapping regions I and II. By choosing  $e_2 = e_2^*$ , agent 2 can cause  $S(e)$  to not intersect region II. For  $R$  sufficiently high, this will be optimal for agent 2. Hence, we can restrict attention to case (i). Now, to have  $S(e)$  contained in region I under the conditions of (2a), it must be that  $e_p > e_p^*$ . But then it would be optimal for agent 1 to reduce  $e_1$  below  $e_1^*$ , since she can do so and keep  $S(e)$  entirely in region I. It follows that no equilibrium choices satisfy the conditions of (2a). Case (2b) is rejected by the analogous argument.

### Case 3

Since both agents are shirking, the principal can choose  $e_p = 0$  and cause  $S(e)$  to overlap or be contained in region IV. For  $R$  sufficiently high, the principal prefers this to any  $e_p \geq e_p^*$ . Hence, no equilibrium has values satisfying (3).

### Case 4

Since  $e_i \geq e_i^*$ ,  $i=1,2$ ,  $S(e)$  is contained in the union of regions I, II, and III for any  $e_p$ . Since the principal's total wage bill is constant over these regions, his best response has  $e_p \geq e_p^*$  by (A3.1).

### Case 5

Choices satisfying (5a) would place at least part of  $S(e)$  in region II and any remaining part in region I. (To verify the latter, from  $e = e^*$ , first reduce  $e_p$  below  $e_p^*$ , then  $e_2$  below  $e_2^*$ , and consider the effect of increasing  $e_1$ .) By (A3.3) and (A3.1), there exists a value  $e_2 = \tilde{e}_2$ , satisfying:  $f^2(\tilde{e}_2, e_p, \theta_{\min}^2) = f^2(e_2^*, e_p^*, \theta_{\min}^2)$  with  $\tilde{e}_2(e_p) \in (e_2^*, \hat{e}_2)$ . This choice

by agent 2 would place  $S(e)$  entirely in region I, avoiding the chance of the penalty R, and is, consequently, agent 2's best response. Its inconsistency with (5a) shows such choices cannot make up an equilibrium. Case (5b) is rejected analogously.

#### Case 6

Given  $e_i < e_i^*$ ,  $i = 1, 2$ , since the principal collects a penalty from each agent in region IV, either choosing some  $e_p < e_p^*$  that places  $S(e)$  entirely in region IV or choosing  $e_p = 0$  with  $S(e)$  partially in region IV would be optimal. But, say agent 1, could choose a higher effort ( $e_1 = e_1^*$  suffices) that would cause  $S(e)$  to lie entirely in the union of regions I and II. Hence, equilibrium cannot have  $S(e)$  overlap region IV. Further, it cannot have  $S(e)$  overlap regions II or III. If  $S(e)$  overlaps region II (III), then, by (A3.3), agent 2 (1) can increase effort causing  $S(e)$  to move out of region II (III) and avoid the penalty. Neither can  $S(e)$  overlap IB. Either agent could gain by increasing effort to move  $S(e)$  into the region (or further into the region) where that agent is rewarded. Hence,  $S(e)$  must lie in region IA completely. But this contradicts the conditions of case (6).

## APPENDIX C

### PROOF OF NONNEGATIVITY OF MULTIPLIERS IN A SPECIAL CASE

Suppose that  $\mu < 0$ ,  $\eta < 0$ , and  $e_1$  and  $e_p$  are strategic substitutes or strategic independents. From the first order conditions of [PA-1],

$$\begin{aligned} \int_0^{\infty} [q_1 - w_1(q_1)] g_{e_1}(q_1|e_1, e_p) dq_1 &= -\eta \int_0^{\infty} [q_1 - w_1(q_1)] g_{e_1 e_p}(q_1|e_1, e_p) dq_1 \\ -\mu \left[ \int_0^{\infty} U^1(w_1(q_1)) g_{e_1 e_p}(q_1|e_1, e_p) dq_1 - d_{e_1}^1(e_1) \right] &< 0 \end{aligned} \quad (C.1)$$

$$\begin{aligned} \lambda \int_0^{\infty} U^1(w_1(q_1)) g_{e_p}(q_1|e_1, e_p) dq_1 &= -\mu \int_0^{\infty} U^1(w_1(q_1)) g_{e_1 e_p}(q_1|e_1, e_p) dq_1 \\ -\eta \left[ \int_0^{\infty} (q_1 - w_1(q_1)) g_{e_1 e_p}(q_1|e_1, e_p) dq_1 - c_{e_p} e_p(e_p) \right] &< 0 \end{aligned} \quad (C.2)$$

$$\int_0^{\infty} U^1(w_1(q_1)) g_{e_1}(q_1|e_1, e_p) dq_1 = d_{e_1}^1(e_1) > 0 \quad (C.3)$$

$$\int_0^{\infty} (q_1 - w_1(q_1)) g_{e_p}(q_1|e_1, e_p) dq_1 = c_{e_p}(e_p) > 0 \quad (C.4)$$

From (C.1) and (C.3),

$$\int_0^{\infty} (q_1 - w_1(q_1) - U^1(w_1(q_1)) g_{e_1}(q_1|e_1, e_p) dq_1 < 0 \quad (C.5)$$

From (C.2), and (C.4),

$$\int_0^{\infty} (q_1 - w_1(q_1) - U^1(w_1(q_1)) g_{e_p}(q_1|e_1, e_p) dq_1 > 0 \quad (C.6)$$

Evidently, (C.5) and (C.6) contradicts. Therefore, both  $\mu < 0$   $\eta < 0$  is not possible.

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## BIOGRAPHICAL SKETCH

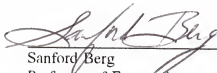
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



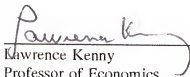
David Sappington, Chair  
Lanzilotti-McKethan Professor  
of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Sanford Berg  
Professor of Economics

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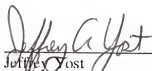
Lawrence Kenny  
Professor of Economics

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Richard Romano  
Associate Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Jeffrey A. Post  
Assistant Professor of Accounting

This dissertation was submitted to the Graduate Faculty of the Department of Economics in the College of Business Administration and to the Graduate School and was accepted as partial fulfillment of the requirement for the degree of Doctor of Philosophy.

August 1994

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Dean, Graduate School